WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015
Round 1: Arithmetic

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. If a ✪ $b = a^2 + \dfrac{a}{b}$ and a❖$b = b$ ✪ a, then find the value of

   9 ✪ (2 ❖ 4).

2. Evaluate $\dfrac{-\left(-2(sm)^t\left(\dfrac{1}{sm}\right)^{t+1}\right)^{-2}}{\left(5s^2m^4\right)^{-3}}$ for $m = \dfrac{1}{2}$, s = 4, and $t = 5$.

3. Find the value of the following infinite continued fraction:

$$\cfrac{3}{3+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3+\ddots}}}}$$

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

(3 pts.) 3. ____________________

Varsity Meet 1 – October 7, 2015
Round 2: Algebra 1

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. For $a \neq b$, solve the following equation for x: $a(a-x) = b\,(b-x)$

2. A system of equations that graphs as parallel lines has no solution. This is called an "inconsistent" system of equations. Determine the value(s) of k that make the following system inconsistent:

$$\begin{cases} 9x + ky = 2 \\ -kx - y = 4 \end{cases}$$

3. Solve the equation $6^{2a+3b} = \left(\frac{9}{16}\right)\left(2^{-a-5b}\right)$ for integers a and b. Write the answer as an ordered pair (a, b).

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

(3 pts.) 3. (__________,__________)

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015
Round 3: Set Theory

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. Let the set R be the range of $f(x) = 3x^2 - 15x + 20$ when the domain is $\{1, 2, 3, 4\}$. Determine R.

2. Determine the number of elements in $A \cap (B \cup C)$ if A, B, and C are as follows:

$$\begin{cases} A = \{1,\ 2,\ 3, \ldots 2014, 2015\} \\ B = \{\textit{all even numbers}\} \\ C = \{\textit{all perfect squares}\} \end{cases}$$

3. If a two-digit number is squared then reduced by the square of the number formed by reversing the digits of the original number, which two, positive, prime factors will always divide the result?

ANSWERS

(1 pt.) 1. {________________________}

(2 pts.) 2. ________________________

(3 pts.) 3. ________________________

WORCESTER COUNTY MATHEMATICS LEAGUE

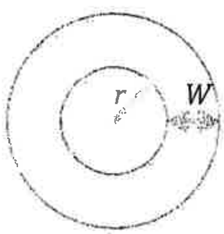
Varsity Meet 1 – October 7, 2015
Round 4: Measurement

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. The area of a circle increases to 121% of its original area, what is the perimeter, P, of the larger circle in terms of the radius, r, of the smaller circle?

2. The lengths of the bases of an isosceles trapezoid are 14 cm and 22 cm. If the base angles are 60°, find the area of the trapezoid in cm^2.

3. From a circle, C_1, of diameter d, a smaller concentric circle, C_2 with radius r is removed. The remaining area of Circle C_1, the region between two concentric circles is called an annulus. The area of the annulus is four times the area of C_2. What is the width W, of the annulus strictly in terms of r?



ANSWERS

(1 pt.) 1. ______________________ units

(2 pts.) 2. ______________________ cm^2

(3 pts.) 3. ______________________ units

Varsity Meet 1 – October 7, 2015
Round 5: Polynomial Equations

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. Find $\dfrac{ABC}{A+B+C}$ if $(x+A)(x+B)(x+C) = x^3 + 4x^2 - 7x + 8$.

2. Find k if $x - 2$ is a factor of $3x^4 + kx^2 - 8$

3. Find $|a - b|$ if $ab = -170$ and $a + b = 29$.

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________

(3 pts.) 3. ____________________

Varsity Meet 1 – October 7, 2015
Team Round

All answers must be in simplest exact form, and be written on the separate team answer sheet

1. Evaluate $x^x - \dfrac{x^2}{\sqrt{x}} + \dfrac{1}{x}$ for $x = 4$.

2. Simplify $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}+\frac{4}{5}}$.

3. Solve for x: $\dfrac{-17}{x+4} + \dfrac{3x}{x-2} = \dfrac{36}{x^2+2x-8}$.

4. The height (in feet) of a projectile launched vertically is a function of time t, in seconds, is described by $h(t) = -16t^2 + 96t + 256$. How many more feet did it travel from 5 to 6 seconds than it traveled from 3 to 4 seconds?

5. Using only the operations of addition, subtraction, multiplication, division, exponents and parentheses, write the numerical expression for the largest number that may be formed by using each of the digits 0, 1, 2, and 3 exactly once.

6. Farmer Jack grows apples, corn and pumpkins. Of his 50 scarecrows, 6 will be placed in the field with only apples, 1 in the field that has both apples and corn, 4 in the field with just pumpkins and corn, and 4 in the field with just apples and pumpkins. There are a total of 31 scarecrows among the corn fields, there are a total of 19 scarecrows among the pumpkin fields and 13 in the apple fields. What is the probability of randomly selecting one of the scarecrows in the field of only corn?

7. $f(x) = ax^2 + bx + c$. The sum of the roots of $f(x)$ is 4 and the difference of the roots of $f(x)$ is 12. Find $\dfrac{c}{ab}$ if a and b are relatively (mutually) prime.

8. A large cube has a surface area of 216 cm^2. What is the total surface area of two smaller cubes whose edges are half the length of the larger cube?

9. When $x^2 + 3x - 10$ is divided by $x - a$, the remainder is -6. If the divisor were to increase by 1, the remainder increases by 6. Determine the value of a.

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015
Team Round Answer Sheet

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM, AND BE WRITTEN ON THIS TEAM ANSWER SHEET.

(2 points each)

1. ______________________________

2. ______________________________

3. ______________________________

4. ____________________________feet

5. ______________________________

6. ______________________________

7. ______________________________

8. ______________________________ cm^2

9. _________________________

Varsity Meet 1 – October 7, 2015
Answers

Round 1: Arithmetic

1. 85.5 or 85½
2. –125
3. $\frac{-3+\sqrt{13}}{2}$

Round 2: Algebra 1

1. $a+b$
2. ±3
3. (4, –2)

Round 3: Set Theory

1. {2, 8}
2. 1029
3. 3 and 11 in either order

Round 4: Measurement

1. $2.2r\pi$ units
2. $72\sqrt{3}$ cm^2
3. $\sqrt{5}r-r$ or $r\left(\sqrt{5}-1\right)$ units

Round 5: Polynomial Equations

1. 2
2. –10
3. 39

Team Round

1. 248.25 or 248¼
2. $\frac{70}{93}$
3. $-\frac{1}{3}$
4. 64 feet
5. $2^{\left(3^{10}\right)}$ or $2^{3^{10}}$
6. 48% or .48 or $\frac{24}{50}$
7. 8
8. 108 cm^2
9. 1

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015 Coaches Booklet

Round 1: Arithmetic

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. If $a \,✪\, b = a^2 + \dfrac{a}{b}$ and $a ❖ b = b \,✪\, a$, then find the value of

$$9 \,✪\, 2 ❖ 4.$$

Solution:

$$9 \,✪\, 2 ❖ 4 = 9^2 + \frac{9}{2} ❖ 4$$
$$= 81 + 4.5 ❖ 4$$
$$= 85.5 ❖ 4$$
$$= 4 \,✪\, 85.5$$
$$= 4^2 + \frac{85.5}{4}$$
$$= 16 + 21.375$$
$$\boxed{= 37.375}$$

if the problem is 9 ✪ (2 ❖ 4) then

$$9 \,✪\, (2 ❖ 4) = 9 \,✪\, 4 \,✪\, 2$$
$$= 9 \,✪\, 4^2 + \frac{4}{2}$$
$$= 9 \,✪\, 16 + 2$$
$$= 9 \,✪\, 18$$
$$= 9^2 + \frac{9}{2}$$
$$= 81 + 4.5$$
$$\boxed{= 85.5}$$

2. Evaluate $\dfrac{-\left(-2(sm)^t\left(\dfrac{1}{sm}\right)^{t+1}\right)^{-2}}{\left(5s^2m^4\right)^{-3}}$ for $m = \dfrac{1}{2}$, $s = 4$, and $t = 5$.

Solution:

$$\frac{-\left(-2(sm)^t\left(\frac{1}{sm}\right)^{t+1}\right)^{-2}}{\left(5s^2m^4\right)^{-3}}=\frac{-\left(-2\left(4\left(\frac{1}{2}\right)\right)^5\left(\frac{1}{4\left(\frac{1}{2}\right)}\right)^{5+1}\right)^{-2}}{\left(5(4^2)\left(\frac{1}{2}\right)^4\right)^{-3}}$$

$$=\frac{-\left(-2(2)^5\left(\frac{1}{2}\right)^6\right)^{-2}}{\left(5(2^4)\left(\frac{1}{2}\right)^4\right)^{-3}}$$

$$=\frac{-\left(-2\left(\frac{1}{2}\right)\right)^{-2}}{(5)^{-3}}$$

$$=\frac{-(-1)^{-2}}{(5)^{-3}}$$

$$=\frac{-(5)^3}{(-1)^2}$$

$$\boxed{=-125}$$

Alternative method to solve in terms of *m*, *s*, and *t* then substitute in

$$\frac{-\left(-2(sm)^t\left(\frac{1}{sm}\right)^{t+1}\right)^{-2}}{\left(5s^2m^4\right)^{-3}}=\frac{-\left(-2\left(\frac{1}{sm}\right)\right)^{-2}}{\left(5s^2m^4\right)^{-3}}$$

$$=\frac{-\left(5s^2m^4\right)^3}{\left(-2\left(\frac{1}{sm}\right)\right)^2}$$

$$=\frac{-\left(5(4^2)\left(\frac{1}{2}\right)^4\right)^3}{\left(-2\left(\frac{1}{2}\right)\right)^2}$$

$$=\frac{-(5(1))^3}{(-1)^2}$$

$$\boxed{=-125}$$

3. Find the value of the following infinite continued fraction:

$$\cfrac{3}{3+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3+\ddots}}}}$$

Solution:

Let $x = \cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{\ddots}}}$

$$3x = \frac{3}{3+x}$$

$$x = \frac{1}{3+x}$$

$x(3+x) = 1$
$3x + x^2 = 1$
$x^2 + 3x - 1 = 0$

use the quadratic to solve for the positive root of the equation:

$$x = \frac{-3+\sqrt{3^2-4(1)(-1)}}{2(1)}$$

$$= \frac{-3\pm\sqrt{9+4}}{2}$$

$$\boxed{= \frac{-3\pm\sqrt{13}}{2}}$$

Round 2: Algebra 1
All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. For $a \neq b$, solve the following equation for x: $a(a-x) = b(b-x)$

Solution:
$a(a-x) = b(b-x)$
$a^2 - ax = b^2 - bx$
$a^2 - b^2 = ax - bx$
$a^2 - b^2 = (a-b)x$
Solve for x remembering that $a \neq b$

$$x = \frac{a^2-b^2}{a-b}$$

$$= \frac{(a-b)(a+b)}{a-b}$$

$$\boxed{= a+b}$$

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

$A \cap B$ is the set of even numbers from 1 to 2015

the number of elements is $\frac{2015-1}{2} = 1007$

$A \cap C$ is the number of perfect squares less than 2015

45^2 is 2025, so 44^2 is 1936

There are 44 perfect squares from 1 to 2015

22 of these perfect squares are also even.

$A \cap B \cap C = 22$

$A \cap B + A \cap C - A \cap B \cap C$

$A \cap (B \cup C) = 1007 + 44 - 22 = \boxed{1029}$

3. If a two-digit number is squared then reduced by the square of the number formed by reversing the digits of the original number, which two, positive, prime factors will always divide the result?

Solution:

Method 1. Pick a number such as 12

$|12^2 - 21^2| = |21^2 - 12^2| = 441 - 144 = 297$

Find the factors of 397 = $27 \times 11 = 3^2 \cdot 11$

3 & 11 are positive prime factors

Method 2.

Let x be a two digit number such that

$x = a10 + b$ reversing the digits of gives $10b + a$

Square both and subtract

$|(10a+b)^2 - (10b+a)^2|$

$= |100a^2 + 20ab + b^2 - (100b^2 + 20ab + a^2)|$

$= |99a^2 - 99b^2|$

$= 99|a^2 - b^2|$

from 99 there are two prime factors $\boxed{3, 11}$

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015 Coaches Booklet

Round 4: Measurement

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. The area of a circle increases to 121% of its original area, what is the ~~perimeter~~ Circumference, P, of the larger circle in terms of the radius, r, of the smaller circle?

Solution:

Let r = original circle's radius (in units)

$Area_{original} = \pi r^2$

$Area_{new} = 1.21 \pi r^2$

$= (1.1r)^2 \pi$

new radius is $1.1r$

$C = \pi d$ or $2\pi r \Rightarrow C = 2.2\pi r$ units

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2. The lengths of the bases of an isosceles trapezoid are 14 cm and 22 cm. If the base angles are 60°, find the area of the trapezoid in cm^2.

$A_{Trapezoid} = \frac{1}{2}(b_1 + b_2)h$ $\quad b_1 + b_2 = 14 + 22 = 36$

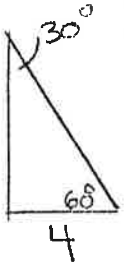
must solve for h $\quad$ trapezoid bases

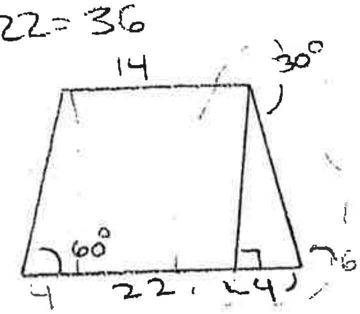
special right triangle $\quad 22 - 14 = 8$

30-60-90 $\quad 8 \div 2 = 4$

$1 - \sqrt{3} - 2$ $\quad$ So the short side

$4 - 4\sqrt{3} - 8$ $\quad$ of a 30-60-90 triangle is 4





so the height of the trapezoid is $4\sqrt{3}$

$A = \frac{1}{2}(22 + 18) \cdot 4\sqrt{3}$

$= \frac{1}{2}(36)4\sqrt{3}$

$= \boxed{72\sqrt{3} \text{ cm}^2}$

2. A system of equations that graphs as parallel lines has no solution. This is called an "inconsistent" system of equations. Determine the value(s) of k that make the following system inconsistent:

$$\begin{cases} 9x+ky=2 \\ -kx-y=4 \end{cases}$$

Solution:

The system will have no solution if the two lines share a slope. We must first put each line into $y =$ form:

$$\begin{cases} ky = 2-9x \\ -y = 4+kx \end{cases}$$

$$\begin{cases} y = \frac{2}{k} - \frac{9x}{k} \\ y = -4-kx \end{cases}$$

$$\begin{cases} y = -\frac{9x}{k} + \frac{2}{k} \\ y = -kx-4 \end{cases}$$

Now we equate slopes and solve for k:

$-\frac{9}{k} = -k$

$\frac{9}{k} = k$

$9 = k^2$

$\boxed{k = \pm 3}$

3. Solve the equation $6^{2a+3b} = \left(\frac{9}{16}\right)\left(2^{-a-5b}\right)$ for integers a and b. Write the answer as an ordered pair (a, b).

Solution:

We need to break each portion of the equation into its prime factors, combine like terms, and equate exponents to determine the system of equations that will determine a and b:

$6^{(2a+3b)} = (\frac{9}{16})2^{(-a-5b)}$

$(3*2)^{(2a+3b)} = (3^2)(2^{-4})2^{(-a-5b)}$

$3^{(2a+3b)}2^{(2a+3b)} = 3^2 2^{(-a-5b-4)}$

$\begin{cases} 2a+3b=2 \\ 2a+3b=-a-5b-4 \end{cases}$ We may solve by substitution or cancellation here. Let's proceed with substitution. Notice that the first equation appears exactly in its original form in the second equation. Thus:

$$\begin{cases} 2 = -a - 5b - 4 \end{cases}$$

$$\begin{cases} 2a + 3b = 2 \\ a + 5b = -6 \end{cases}$$

We proceed with cancellation:

$$\begin{cases} 2a + 3b = 2 \\ 2a + 10b = -12 \end{cases}$$ Subtracting the first equation from the second yields:

-7b=14

b=-2

And substituting the value for b into the first equation gives the value of a:

2a+3(-2)=2

2a-6=2

2a=8

a=4

So the answer is $(a, b) = \boxed{(4, -2).}$

Round 3: Set Theory
All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. Let the set R be the range of $f(x) = 3x^2 - 15x + 20$ when the domain is $\{1, 2, 3, 4\}$. Determine R.

Solution: $f(1) = 3(1)^2 - 15(1) + 20 = 3(1) - 15 + 20 = -12 + 20 = 8$

$f(2) = 3(2)^2 - 15(2) + 20 = 3(4) - 30 + 20 = 12 - 10 = 2$

$f(3) = 3(3)^2 - 15(3) + 20 = 3(9) - 45 + 20 = 27 - 25 = 2$

$f(4) = 3(4)^2 - 15(4) + 20$=3(16)-60+20=48-40=8

So $\boxed{R = \{2, 8\}.}$

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2. Determine the number of elements in $A \cap (B \cup C)$ if A, B, and C are as follows:

$$\begin{cases} A = \{1,\ 2,\ 3, \ldots 2014,\ 2015\} \\ B = \{\textit{all even numbers}\} \\ C = \{\textit{all perfect squares}\} \end{cases}$$

Solution:

7. $f(x) = ax^2 + bx + c$. The sum of the roots of $f(x)$ is 4 and the difference of the roots of $f(x)$ is 12. Find $\frac{c}{ab}$ if a and b are relatively (mutually) prime.

Solution:

Let g, h be roots of $f(x) = ax^2 + bx + c$

$g + h = 4$
$g - h = 12$
so $2g = 16$ $g = 8$ $h = -4$

$f(x) = (x - (-4))(x - 8) = (x+4)(x-8) = x^2 - 4x - 32$

so $a = 1$ $b = -4$ $c = -32$

so $\frac{c}{ab} = \frac{-32}{(1)(-4)} = \frac{-32}{-4} = \boxed{8}$

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8. A large cube has a surface area of 216 cm^2. What is the total surface area of two smaller cubes whose edges are half the length of the larger cube?

Solution:

Larger cube surface area is $6 \cdot s^2$ where s is the length of edge of the larger cube

$s^2 = \frac{216}{6} = 36$ so $s = 6$

The smaller cubes have side length $\frac{6}{2} = 3$

The surface area of a cube with sidelength 3 is $6 \cdot 3^2 = 6 \cdot 9 = 54$. 2 of them have total surface area $\boxed{108 \text{ cm}^2}$

9. When $x^2 + 3x - 10$ is divided by $x - a$, the remainder is –6. If the divisor were to increase by 1, the remainder increases by 6. Determine the value of a.

Solution:

Use synthetic division

$a\rfloor$	1	3	−10
		a	a^2+3a
	1	$3+a$	$a^2+3a-10$

$a^2+3a-10=6$

so $a^2+3a-4=0$

$(a-1)(a+4)=0$

$a=1$ or $a=-4$

so check $a=2$ or $a=-3$

$2\rfloor$	1	3	−10
		2	10
	1	5	0

$0=-6+6$

2 is a divisor

$3\rfloor$	1	3	−10
		3	18
	1	6	8

divisor not

$8 \neq -6+6$

$\boxed{a=1}$

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015 Coaches Booklet

Team Round

All answers must be in simplest exact form, and be written on the separate team answer sheet

1. Evaluate $x^x - \frac{x^2}{\sqrt{x}} + \frac{1}{x}$ for $x = 4$.

Solution: $4^4 - \frac{4^2}{\sqrt{4}} + \frac{1}{4}$

$= 64 - \frac{16}{2} + \frac{1}{4}$

$= 64 - 8 + \frac{1}{4}$

$= \boxed{56\frac{1}{4}} = 56.25$

2. Simplify $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}+\frac{4}{5}}$.

Solution: $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}+\frac{4}{5}} = \dfrac{\frac{3}{6}+\frac{4}{6}}{\frac{15}{20}+\frac{16}{20}} = \dfrac{\frac{7}{6}}{\frac{31}{20}} = \frac{7}{6} \div \frac{31}{20} = \frac{7}{6} \cdot \frac{20}{31} = \boxed{\frac{70}{93}}$

3. Solve for x: $\frac{-17}{x+4} + \frac{3x}{x-2} = \frac{36}{x^2+2x-8}$.

$x \neq 2$ and $x \neq -4$

Solution: $-17(x-2) + 3x(x+4) = 36$

$-17x + 34 + 3x^2 + 12x = 36$

$= 3x^2 - 5x - 2$

$= (3x+1)(x-2)$

$x = -\frac{1}{3}$ or $x = 2$ but $x \neq 2$ so $\boxed{x = -\frac{1}{3}}$

no division by zero

4. The height (in feet) of a projectile launched vertically is a function of time t, in seconds, is described by $h(t) = -16t^2 + 96t + 256$. How many more feet did it travel from 5 to 6 seconds than it traveled from 3 to 4 seconds?

$h(3) = -16(3^2) + 96(3) + 256 = -16(9) + 288 + 256 = -144 + 544 = 400$

$h(4) = -16(4^2) + 96(4) + 256 = -16(16) + 384 + 256 = -256 + 384 + 256 = 384$

$h(5) = -16(5^2) + 96(5) + 256 = -16(25) + 480 + 256 = -400 + 736 = 336$

$h(6) = -16(6^2) + 96(6) + 256 = -16(36) + 576 + 256 = -576 + 576 + 256 = 256$

$h(3) - h(4) = 16$ and $h(5) - h(6) = 80$. So the projectile traveled 64 more feet from 5 to 6 seconds than from 3 to 4.

5. Using only the operations of addition, subtraction, multiplication, division, exponents and parentheses, write the numerical expression for the largest number that may be formed by using each of the digits 0, 1, 2, and 3 exactly once.

$\boxed{2^{(3^{10})}} > 3^{(2^{10})} = 3^{1024} > (3^2)^{10} > (2^3)^{10} > 10^{3^2} > 10^{2^3}$

(10 digits) (10 digits) (9 digits)

this can be checked using a log ... of logs to determine how many digits each number has

$\log 2 = 0.301$ $3^{10} = 59049$ ← (looked at but ... 1024)

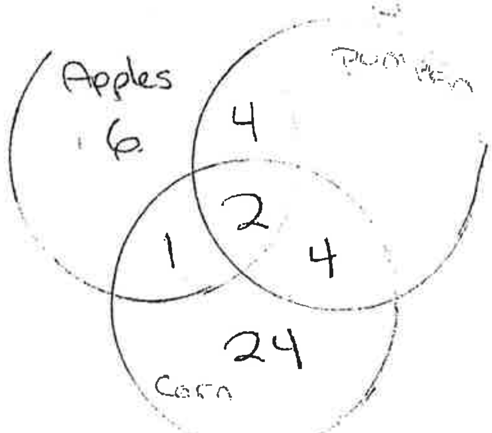
so ... $.301 \times 59{,}049 \approx 17{,}773$ digits

where $\log 3 = 0.477$ $2^{10} = 1024 \approx 488$ digits

6. Farmer Jack grows apples, corn and pumpkins. Of his 50 scarecrows, 6 will be placed in the field with only apples, 1 in the field that has both apples and corn, 4 in the field with just pumpkins and corn, and 4 in the field with just apples and pumpkins. There are a total of 31 scarecrows among the corn fields, there are a total of 19 scarecrows among the pumpkin fields and 13 in the apple fields. What is the probability of randomly selecting one of the scarecrows in the field of only corn?

Solution:

Use a Venn diagram



Apples
$13 - (6+4+1) = 2$

There are 2 scarecrows in the field with apples corn & pumpkins

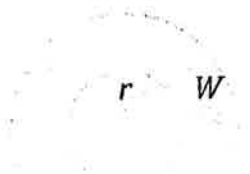
Corn
$31 - (1+2+4) = 24$

so probability of randomly selecting a corn only scarecrow is $\boxed{\frac{24}{50}} = .48 = 48\%$

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015 Coaches Booklet

3. From a circle, C_1, of diameter d, a smaller concentric circle, C_2 with radius r is removed. The remaining area of Circle C_1, the region between two concentric circles is called an annulus. The area of the annulus is four times the area of C_2. What is the width W, of the annulus strictly in terms of r?



$Area_{Circle} = \pi r^2$

$Area_{Circle, C_1} = \pi$

Circle C_1
diameter = d
radius = $d/2$
Area = $\pi \left(\frac{d}{2}\right)^2$
=

Circle C_2
diameter = $2r$
radius = r
Area = πr^2

Area Circle C_1 = Area of Annulus + Area of Circle C_2
$= 4\pi r^2 + \pi r^2$
$= 5\pi r^2$

$radius_{C_1} = \sqrt{5r^2}$
$= \sqrt{5}\, r$

Width of Annulus is $C_1 - C_2$ $(\sqrt{5}\, r - r)$ units
$= r(\sqrt{5} - 1)$ units

WORCESTER COUNTY MATHEMATICS LEAGUE

Varsity Meet 1 – October 7, 2015 Coaches Booklet

Round 5: Polynomial Equations

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Find $\dfrac{ABC}{A+B+C}$ if $(x+A)(x+B)(x+C) = x^3 + 4x^2 - 7x + 8$.

Solution:

Expand $(x+A)(x+B)(x+C)$.

$$x^3 + (A+B+C)x^2 + (AB+AC+BC)x + ABC$$
$$= x^3 + 4x^2 - 7x + 8$$

So $A+B+C = 4$ $\quad ABC = 8$

$$\frac{ABC}{A+B+C} = \frac{8}{4} = \boxed{2}$$

2. Find k if $x-2$ is a factor of $3x^4 + kx^2 - 8$

Solution:

Use synthetic Division

2	3	0	k	0	−8
		6	12	24+2k	48+4k
	3	6	12+k	24+2k	40+4k

2 is a factor when $40 + 4k = 0$

so $\dfrac{4k}{4} = \dfrac{-40}{4}$

$k = -10$

3. Find $|a-b|$ if $ab = -170$ and $a + b = 29$.

Solution:

Find the factors of 170

$1 \cdot 170 = 2 \cdot 85 = 5 \cdot 34 = 10 \cdot 17$

Find the factor pair that has a difference of 29

$5 \cdot 34$ if $a = -5$ & $b = +34$ or if $a = 34$ $b = -5$

or $|34 - (-5)|$

... $24| - \boxed{39}$ ... $= 39$