

**Worcester County Mathematics
League**

WOCOMAL Varsity Meet #4

Coaches' Booklet

April 7, 2004

April 7, 2004

WOCOMAL Varsity Meet

Round 1: Elementary Number Theory (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Convert the base five number **22.22** to a base ten number.
2. The GCF (greatest common factor) of 84 and another number, **y**, is 42. Their LCM (least common multiple) is 252. Determine the value of **y**.
3. What is the last digit of the product you obtain by multiplying the first 2002 odd prime numbers together?

ANSWERS

- (1 pt.) 1. _____
- (2 pts.) 2. _____
- (3 pts.) 3. _____

Hudson, Tantasqua, Algonquin

April 7, 2004

WOCOMAL Varsity Meet

Round 2: Algebra 1 (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $3x + y = a$ and $4x - y = b$, then determine the value of y in terms of a and b .

2. Determine the value of m so that $(x-3)$ is a factor of $4x^2 - 6x + m$.

3. The membership ratio of adults to minors at the Racquet Club is **3:2**. A membership drive was held and 150 minors joined the club. The new ratio of adults to minors is now **2:3**. Determine the new **total** membership of the Racquet Club.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

St. John's, Bromfield, Worcester Academy

April 7, 2004

WOCOMAL Varsity Meet

Round 3: Geometry (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. Determine the perimeter of the quadrilateral formed by joining the midpoints of the sides of the quadrilateral whose diagonals are 10 inches and 8 inches.

2. Determine the area of an equilateral triangle inscribed in a circle of radius 12 cm.

3. A “4 x 4 x 4” cube is made from 32 white unit cubes and 32 black unit cubes. What is the largest possible percent of black surface area?

ANSWERS

(1 pt.) 1. _____ in.

(2 pts.) 2. _____ cm²

(3 pts.) 3. _____

Southbridge, Auburn

April 7, 2004

WOCOMAL Varsity Meet

Round 4: Logarithms, Exponents, and Radicals (NO CALCULATORS)

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM.

1. Find ALL values of x which satisfy: $2^{(x+2)} = (\frac{1}{2})^x$.
2. If $\log_3 (\log_{27} (\log_8 2x)) = -1$, determine the value of x .
3. Let $f(x) = \sqrt{3^x}$. Find the **exact** value of h if $f(x+h) = 2f(x)$ and write your answer in the form of $\log_b a$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Nipmuc, Hudson, Worcester Academy

April 7, 2004

WOCOMAL Varsity Meet

Round 5: Trigonometry (OPEN)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1. If $\tan x + \cot x = 144/25$, determine the numerical value of $1/\tan x + 1/\cot x$.

2. If you simplify the following: $\frac{\sin^3 x + \cos^3 x}{1 - \sin x \cos x}$ write your answer

in the form $A + B$ with **NO** fractions.

3. If $2(\sec^2 x - 1)\sin x \cos x (1 - 2\sin^2 x)$ is re-written in the form:

$$A(\sin Bx)(\cos Cx)(\tan^2 Dx),$$

What is the value of $(A+B+C+D)$?

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Tahanto, Doherty, Assabet Valley Regional

April 7, 2004
TEAM ROUND

WOCOMAL Varsity Meet

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON THE SEPARATE TEAM ANSWER SHEET. (2 points each)

1. What is the smallest natural number (1,2,3,4,5,...) that is divisible by each of the first ten natural numbers?
2. If $x = 1 + 2^p$ and $y = 1 + 2^{-p}$, then express y in terms of a single fraction involving x .
3. In a triangle whose vertices are: (3, 8), (10, -6), (-1, -8), determine the coordinates for the point of intersection of the three medians of the triangle.
4. Solve: $\log_2 x - \log_x 16 = 3$
5. If $\sin x = p$, find, in terms of p only, an expression for $\sin 3x$.
6. What is the LCM in base five of: 10100_{two} and 101010_{two} ?
7. Each side of triangle XYZ is 20 units long. For how many points O on side YZ is the length XO an integer?
8. If $a^2 - bc = 7$, $b^2 + ac = 7$, $c^2 + ab = 7$, then determine the value of $a^2 + b^2 + c^2$.
9. The midpoints of the sides of a triangle are (1,1), (4,3), (3,5). Find the area of the triangle.

Burncoat, Bromfield, Assabet Valley, Worcester Academy, Shepherd Hill

April 7, 2004

WOCOMAL Varsity Meet ANSWERS

Round 1: Elementary Number Theory

1. (1 pt.) 12.48
 2. (2 pts.) 126
 3. (3 pts.) 5
-

Round 2: Algebra 1-open

1. (1 pt.) $(4a-3b)/7$
 2. (2 pts.) $m = -18$
 3. (3 pts.) 450
-

Round 3: Geometry (OPEN)

1. (1 pt.) 18
 2. (2 pts.) $108\sqrt{3}$
 3. (3 pts.) 75%
-

Round 4: Logarithms, Exponents, and Radicals

1. (1 pt.) $x = -1$
 2. (2 pts.) 256
 3. (3 pts.) $\log_3 4$
-

Round 5: Trigonometry (OPEN)

1. (1 pt.) 144/25
 2. (2 pts.) $\sin x + \cos x$
 3. (3 pts.) 6
-

April 7, 2004

TEAM ROUND (2 pts. Each)

1. 2520
2. $y = x/(x-1)$
3. (4, -2)
4. $x = 16$ or $x = \frac{1}{2}$ (BOTH needed)
5. $3p - 4p^3$ or $p(3-4p^2)$
6. $(3140)_{\text{five}}$
7. 6
8. 14
9. 16

April 7, 2004

WOCOMAL Varsity Meet

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET. (2 points each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

April 7, 2004

WOCOMAL Varsity Meet

TEAM ROUND

School: _____

Team #: _____

Team Members:

1. _____
2. _____
3. _____
4. _____
5. _____

Total Points for Team Round: _____