

March 5, 2008

WOCOMAL Varsity Meet

Round 1: Elementary Number Theory (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. What is the smallest composite number generated by replacing p in $p^2 - p - 1$ with a prime ?

2. $1111_2 + 2222_3 + 3333_4 = \text{______}_5$? [The subscripts are bases.]

3. In a modular number system, $3 \times 13 \equiv 9 \times 10$. Normally, 5×6 would equal 30, but not in this system. What is 5×6 ?

ANSWERS

- (1 pt.) 1. _____
- (2 pts.) 2. _____
- (3 pts.) 3. _____

Burncoat, Assabet Valley, North

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Round 2: Algebra 1 (open)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. In an (x, y) coordinate system, write the equation of the vertical line passing through the point of intersection of $3x + 4y = 1$ and $x + 3y = 7$.
2. A chemist has two alcohol-in-water solutions, one 20% alcohol and the other 50%. She needs a solution that is 45% alcohol. In what ratio should she mix the weaker to the stronger alcohol solutions?
3. The sum of two real numbers is 12; the sum of their reciprocals is 2. The exact answers are of the form $a + b\sqrt{c}$. Write either one of these numbers.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____ to _____

(3 pts.) 3. _____

Bromfield, Millbury, Doherty Memorial

Round 3: Geometry (OPEN) !!! problem values are 2, 2, 2 !!!

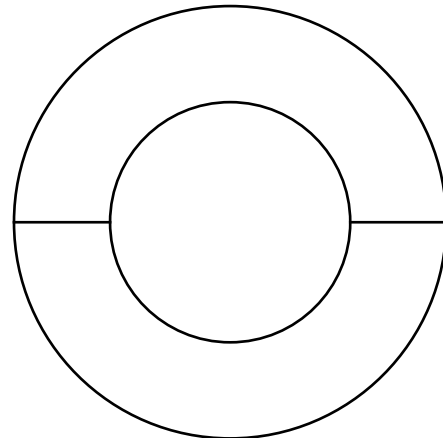
ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

ie. Keep π or simplified radicals.

1. When two circles are externally tangent, the segment between their centers is 11 units long; when internally tangent, the segment between their centers is 5 units long. Compute the difference in the areas of the two circles.

2. Two parallel chords of a circle are both 6 cm. long and 6 cm. apart. In square centimeters find the area of the region inside the circle and between the lines of the chords.

3. Three friends are sharing a circular pizza by cutting it into three equal area parts. Two get more crust by taking the “elbow macaroni” shapes, and one gets more interior by taking a central concentric circular piece, as shown. If the diameter of the original pizza is 20 inches, what is the diameter of the central piece ?



ANSWERS

(2 pts.) 1. _____ *sq. un.*

(2 pts.) 2. _____ *cm²*

(2 pts.) 3. _____ *inches*

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Round 4: Logs, Exponents, and Radicals

(NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Solve $3 \cdot \log_8 x = 5$ for all real solutions.

2. If $N = \log_y 8$ and $y = 2^N$, what are the values of N ?

3. Find the smallest positive integer Z such that $\sqrt{Z+1} - \sqrt{Z} < 0.1$

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____

Shrewsbury, Westborough, Saint John's

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Round 5: Trigonometry (OPEN)

1. Find the simplest form equivalent to this trig expression:

$$\sin^3 \theta + \cos \theta + \sin \theta \cos^2 \theta$$

2. From the stage of a theater the angle of elevation to the first balcony is 19° . From the same point the angle of elevation of the second balcony, 6.3 meters above the first, is 29° . How high above the stage is the first balcony? Round to the nearest tenth of a meter.

3. If $2 + \sqrt{3}$ is an x-solution for the equation $x^2 - (\tan \alpha + \cot \alpha)x - 1 = 0$, compute the value of the product $\cos \alpha \cdot \sin \alpha$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____ *m.*

(3 pts.) 3. _____

Bromfield, Tantasqua, Nashoba

ALL ANSWERS MUST BE IN SIMPLEST FORM AND
ON THE SEPARATE TEAM ANSWER SHEET

(2 points each)

1. June is the 6th month; so the 6th, 12th, etc. days of June are multiples of its month number. How many days in a year are multiples of their month numbers?
2. What are the only two-digit positive integers for which the difference between the integer and the product of its digits is 12 ?
3. Find the area of the plane region defined by $\{(x, y) : |x - y| + |2y| \leq 4\}$.
4. If $\sqrt{x - 3} = \sqrt{x} - \sqrt{2}$, compute the real numerical value of $\sqrt{8x}$.
5. Compute the exact numerical value of $\frac{\cos 15^\circ + \sin 15^\circ}{\cos 15^\circ - \sin 15^\circ}$.
6. You have fifty US coins totaling \$ 1.00 . You drop one of any size down an open drain while tossing the coins in your hand. What is the probability you lost a quarter?
7. Consider triangle ABC with point O interior to the triangle. The following lengths are known: $AC = 29$, $BC = 39$, $OA = 20$, $OB = 24$, and $OC = 21$. To the nearest whole degree, determine the measure of $\angle AOB$.
8. If Mr. Wersted equally distributed the money in his pocket among the students in his class, each would receive \$ 1.26 . If there had been four more students, then each would have received \$ 1.05 . How much money was in his pocket?
9. In a numeration system with a positive integer base, the numbers 104 and 241 are the degree measures of a pair of supplementary angles. What is the base of this numeration system?

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WOCOMAL Varsity Meet ANSWERS

Round 1: No Thy

(1 pt.) 155

(2 pts.) 2400

(3 pts.) 13

Round 2: Algebra 1

(1 pt.) $x = -5$

(2 pts.) 1 to 5 (not reversed)

(3 pts.) $6 + \sqrt{30}$ or $6 - \sqrt{30}$
(either answer)

Round 3: Geometry

(2 pts.) 55π

(2 pts.) $18 + 9\pi$

(2 pts.) $\frac{20\sqrt{3}}{3}$ or $\frac{20}{\sqrt{3}}$

Round 4: Logs, Exps & Rads

(1 pt.) 32

(2 pts.) $\pm\sqrt{3}$ (need both)

(3 pts.) 25

Round 5: Trigonometry

(1 pt.) $\cos \theta + \sin \theta$

(2 pts.) 10.3

(3 pts.) $\frac{\sqrt{3}}{6}$ or 0.289

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TEAM ROUND ANSWERS (2 points each)

1. 90
2. 28 and 39 (need both)
3. 16
4. 5
5. $\sqrt{3}$ or 1.732
6. $\frac{1}{100}$ or 0.01
7. 150° (don't need deg. symbol)
8. \$ 25.20
9. 7

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WOCOMAL Varsity Meet TEAM ROUND ANSWERS

ALL ANSWERS MUST BE IN SIMPLEST FORM and ON THIS SHEET
(2 points each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

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TEAM ROUND

School: _____

Team #: _____

Names of Team Members:

1. _____

2. _____

3. _____

4. _____

5. _____

α β γ δ ε ζ η θ ι κ λ μ ν ξ ο π ρ σ τ υ φ χ ψ ω

Total Points for Team Round: _____