

Worcester County Mathematics League

Freshman Meet 3

March 7, 2018

COACHES' COPY

ROUNDS, ANSWERS, AND SOLUTIONS



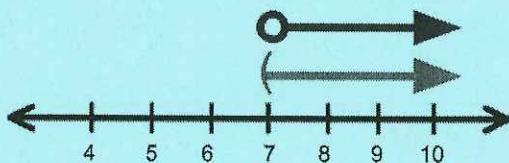
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ANSWER KEY

Round 1:

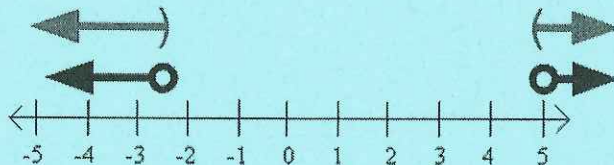
1. (Sulton High)

Note: Either line graphed is an acceptable answer.



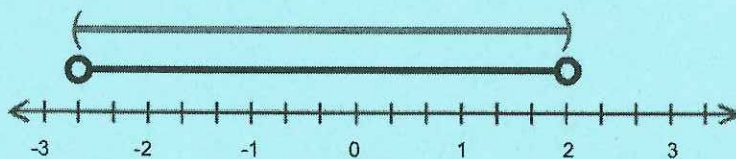
2. (Shrewsbury)

Note: Either line graphed is an acceptable answer.



3. (St. John's)

Note: Either line graphed is an acceptable answer.



Round 2:

1. (Auburn) $x^2 - 3x + 1$
2. (Bromfield) $x = -5$ and $x = -4$
3. (Bartlett) $-2x^3 + 16x^2 - 42x + 188$

Round 3:

1. (Bartlett) $\frac{1}{6}$
2. (Douglas) $\frac{1}{19}$
3. (Hopedale) 2,598,960

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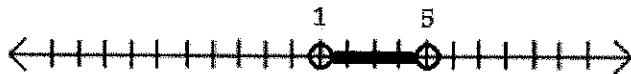


Round 4:

1. (Westborough) 18
2. (Bromfield) $54 - 9\pi$
3. (Bromfield) 546

Team Round:

1. (ASMA Charter School)



OR



2. (Douglas) 83
3. (Bromfield) Daughter: 11, Mother: 36
4. (Hopedale) $A = 28, B = 56, C = 96$
5. (Algonquin) 17
6. (Bartlett) 5,040
7. (Burncoat) $(n - 1)(n + 1)^2$ or $(n - 1)(n + 1)(n + 1)$
8. (Burncoat) 108

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Round 1: Graphing on a Number Line

All answers must be in simplest exact form in the answer section.

NO CALCULATOR ALLOWED

1. Graph the following inequality on a number line:

$$-10(v - 12) < -10(-2v + 9)$$

2. Graph the following: $|\frac{4x}{5} - 1| > 3$

3. Graph all integral values of which x which satisfies:

$$-2x - 4 < -5x + 2 < -2x + 10$$

ANSWERS

(1 pt.) 1.

(2 pt.) 2.

(3 pt.) 3.

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Round 2: Operations on Polynomials

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Divide $(x^4 - 3x^3 - x^2 + 6x - 2)$ by $(x^2 - 2)$

2. Solve for all values of x : $(x + 5)(x + 3) - (2x + 7)(x + 5) = 0$

3. Simplify to a polynomial:

$$6(x - 5)^2 + 4(x + 3)^2 - 2(x - 1)^3$$

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____

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Round 3: Techniques of Counting and Probability

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. What is the probability of rolling “doubles” (2 dice with the same #)?

2. A drawer contains 10 white socks, 8 black socks and 2 red socks. What is the probability that you will randomly pick a white sock followed by a red sock? Write your answer as a simplified fraction.

3. How many different hands of 5 cards each can be dealt from a deck of 52 cards?

ANSWERS

(1 pt.) 1. _____

(2 pt.) 2. _____

(3 pt.) 3. _____

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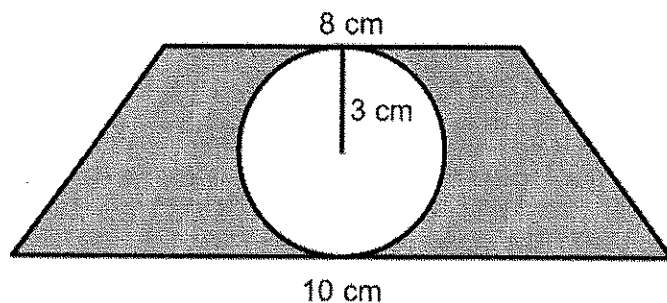
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Round 4: Perimeter, Area, and Volume

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Find the area of a square whose diagonal measures 6 inches.
2. Find the area of the shaded region. Leave your answer in terms of π



3. A rectangle is twice as long as it is wide and has a semicircle on the shorter ends. It also has a 100-meter track around it. Determine to the nearest square meter the area of the region enclosed by the track. Use 3.14 for π .

ANSWERS

(1 pt.) 1. _____ in^2

(2 pt.) 2. _____ cm^2

(3 pt.) 3. _____ m^2



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TEAM ROUND

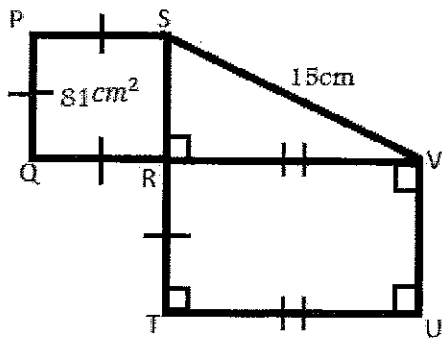
All answers must either be in simplest exact form or rounded to EXACTLY three decimal places, unless stated otherwise (2 points each).

APPROVED CALCULATORS ALLOWED

1. Graph the following inequality on the number line:

$$x^2 - 6x < -5$$

2. At Euclid High School's grade 9, 33 students like volleyball, 34 like softball, 39 like basketball, 20 like volleyball AND softball, 10 like volleyball AND basketball, 8 like softball AND basketball, 3 like all three sports, and 12 like none of these sports. How many students are in Euclid's 9th grade class?
3. A mother is 3 times as old as her daughter will be next year. If the sum of their ages is 47, what are the mother and daughter's present ages?
4. Find the 3 angles of a triangle of $\triangle ABC$ if $\angle A$ is half of $\angle B$, and $\angle A$ is also 12° less than one third of $\angle C$.
5. I have 8 nickels and 3 more than twice as many dimes as quarters. How many dimes do I have if I have \$3.85 in all?
6. How many distinct ways can you arrange the letters in the word "INTERNET"?
7. Factor: $n^3 + n^2 - n - 1$
8. $\square PQRS$ is a square whose area is 81 cm^2 . Find the area of $\square RTUV$





Freshman Meet 3 – SOLUTIONS

Round 1: Graphing on a Number Line

1. Graph the following inequality on a number line:

$$-10(v - 12) < -10(-2v + 9)$$

Solution:

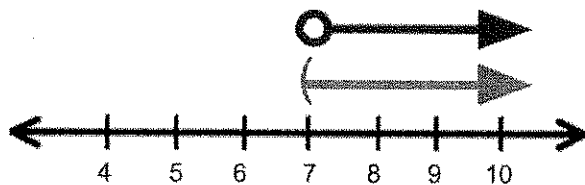
$$-10v + 120 < 20v - 90$$

$$120 < 30v - 90$$

$$210 < 30v$$

$$7 < v \text{ or } v > 7$$

Note: Either line graphed is an acceptable answer.



2. Graph the following: $|\frac{4x}{5} - 1| > 3$

Solution:

$$\frac{4x}{5} - 1 > 3$$

$$\frac{4x}{5} > 4$$

$$4x > 20$$

$$x > 5$$

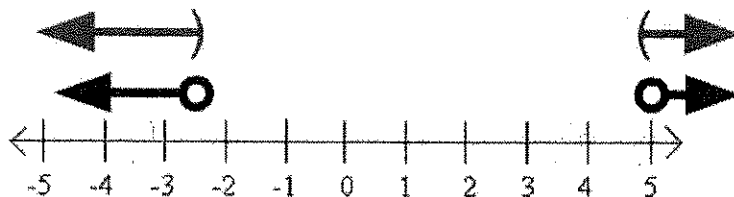
$$\frac{4x}{5} - 1 > -3$$

$$\frac{4x}{5} < -2$$

$$4x < -10$$

$$x < -\frac{5}{2}$$

Note: Either line graphed is an acceptable answer.



3. Graph all integral values of which x which satisfies:

$$-2x - 4 < -5x + 2 < -2x + 10$$

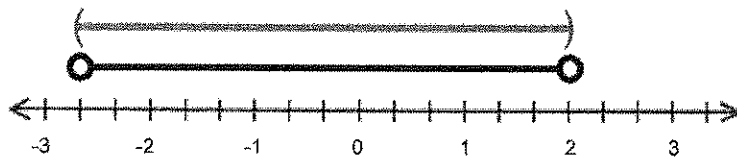
Solution:

$$-4 < -3x + 2 < 10$$

$$-6 < -3x < 8$$

$$2 > x > -\frac{8}{3} \text{ or } -\frac{8}{3} < x < 2$$

Note: Either line graphed is an acceptable answer.





Freshman Meet 3 – SOLUTIONS

Round 2: Operations on Polynomials

1. Divide $(x^4 - 3x^3 + x^2 + 6x - 2)$ by $(x^2 - 2)$

Solution:

$$\begin{array}{r} x^2 - 2 \overline{) \begin{array}{r} x^4 - 3x^3 + x^2 + 6x - 2 \\ -x^4 \\ \hline -3x^3 + x^2 + 6x - 2 \\ +3x^3 \\ \hline x^2 + 6x - 2 \\ -x^2 \\ \hline 6x - 2 \\ -6x \\ \hline -2 \\ +2 \\ \hline 0 \end{array}} \end{array}$$

2. Solve for all values of x : $(x + 5)(x + 3) - (2x + 7)(x + 5) = 0$

Solution:

$$(x + 5)[(x + 3) - (2x + 7)] = 0$$

$$(x + 5)(-x - 4) = 0$$

$$x = -5 \quad x = -4$$

$$-5, -4$$

3. Simplify to a polynomial:

$$6(x - 5)^2 + 4(x + 3)^2 - 2(x - 1)^3$$

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Solution:

$$6(x - 5)^2 + 4(x + 3)^2 - 2(x - 1)^3$$

$$6(x^2 - 10x + 25) + 4(x^2 + 6x + 9) - 2(x^3 - 3x^2 + 3x - 1)$$

$$6x^2 - 60x + 150 + 4x^2 + 24x + 36 - 2x^3 + 6x^2 - 6x + 2$$

$$-2x^3 + 16x^2 - 42x + 188$$



Freshman Meet 3 – SOLUTIONS

Round 3: Techniques of Counting and Probability

1. What is the probability of rolling “doubles” (2 dice with the same #)?

Solution:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\frac{6}{36} \text{ or } \frac{1}{6}$$

The first die does not matter, the second die has a $\frac{1}{6}$ chance of matching the first.

2. A drawer contains 10 white socks, 8 black socks and 2 red socks. What is the probability that you will randomly pick a white sock followed by a red sock? Write your answer as a simplified fraction.

Solution:

$$\left(\frac{10}{20}\right)\left(\frac{2}{19}\right) = \left(\frac{1}{2}\right)\left(\frac{2}{19}\right) = \frac{1}{19}$$

3. How many different hands of 5 cards each can be dealt from a deck of 52 cards?

Solution:

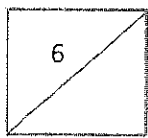
$$\begin{aligned} 52C_5 &= \frac{52!}{5!47!} \\ &= \left(\frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48}{5!}\right) \left(\frac{47!}{47!}\right) & 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= \frac{311,875,200}{120} = 2,598,960 \end{aligned}$$



Freshman Meet 3 – SOLUTIONS
Round 4: Perimeter, Area, and Volume

1. Find the area of a square whose diagonal measures 6 inches.

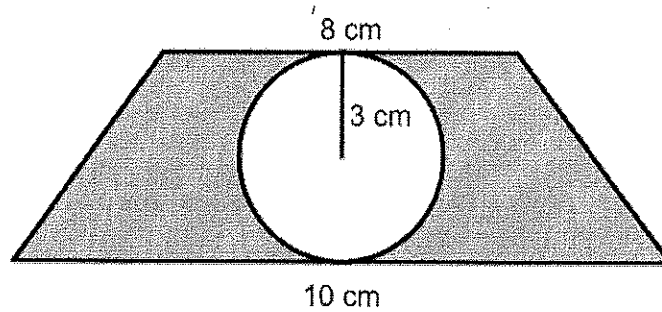
Solution:



$$36 = 2S^2$$
$$18 = S^2$$

Ans. is 18 sq. inches

2. Find the area of the shaded region. Leave your answer in terms of π



Solution:

Area of trapezoid – area of circle

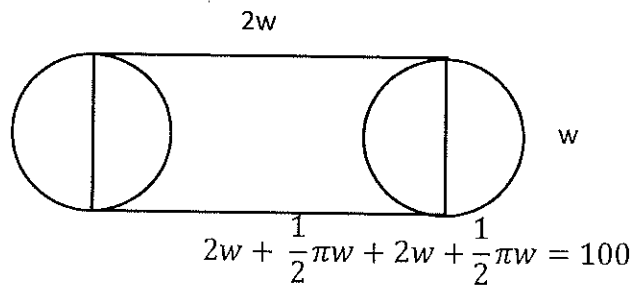
$$\frac{h(b_1 + b_2)}{2} - \pi r^2 = \frac{6(18)}{2} - \pi(9)$$
$$= (54 - 9\pi)cm$$

Ans. $(54 - 9\pi)cm^2$



3. A rectangle is twice as long as it is wide and has a semicircle on the shorter ends. It also has a 100-meter track around it. Determine to the nearest square meter the area of the region enclosed by the track. Use 3.14 for pi.

Solution:



$$w = \frac{100}{4 + \pi}$$

$$area = 2\left(\frac{100}{4 + \pi}\right)^2 + \pi\left(\frac{50}{4 + \pi}\right)^2$$

$$\approx 546m^2$$



Freshman Meet 3 – SOLUTIONS

TEAM ROUND

1. Graph the following inequality on the number line:

$$x^2 - 6x < -5$$

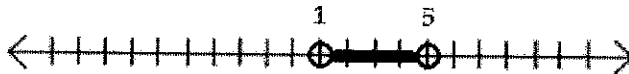
Solution:

$$x^2 - 6x + 9 < -5 + 9$$

$$(x - 3)^2 < 4$$

$$|x - 3| < 2$$

$$1 < x < 5$$

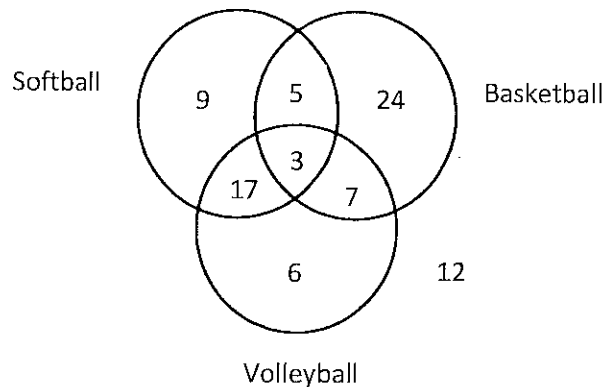


OR



2. At Euclid High School's grade 9, 33 students like volleyball, 34 like softball, 39 like basketball, 20 like volleyball AND softball, 10 like volleyball AND basketball, 8 like softball AND basketball, 3 like all three sports, and 12 like none of these sports. How many students are in Euclid's 9th grade class?

Solution:



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$$3 + (17 + 7 + 5) + (6 + 9 + 24) + 12 = 83 \text{ students}$$

3. A mother is 3 times as old as her daughter will be next year. If the sum of their ages is 47, what are the mother and daughter's present ages?

Solution:

$$\text{Mother now} = 3(x + 1)$$

$$\text{Daughter now} = x$$

$$3(x + 1) + x = 47$$

$$4x + 3 = 47$$

$$4x + 44$$

$$x = 11$$

$$\text{Daughter} = 11$$

$$\text{Mother} = 36$$

4. Find the 3 angles of a triangle of $\triangle ABC$ if $\angle A$ is half of $\angle B$, and $\angle A$ is also 12° less than one third of $\angle C$.

Solution:

$$B = 2A$$

$$C = 3A + 12$$

$$A + B + C = 180$$

$$A + 2A + 3A + 12 = 180$$

$$6A + 12 = 180$$

$$6A = 168$$

$$A = 28$$

$$B = 56$$

$$C = 96$$

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5. I have 8 nickels and 3 more than twice as many dimes as quarters. How many dimes do I have if I have \$3.85 in all?

Solution:

$$\begin{aligned}x &= \# \text{ of quarters} \\2x + 3 &= \# \text{ of dimes} \\8 &= \# \text{ of nickels} \\25(x) + 10(2x + 3) + 5(8) &= 385 \\25x + 20x + 30 + 40 &= 385 \\45x &= 315 \\x &= 7 \\2(7) + 3 &= 17 \text{ dimes}\end{aligned}$$

6. How many distinct ways can you arrange the letters in the word "INTERNET"?

Solution:

$$\begin{aligned}&\frac{8 \text{ letters}}{2 \text{ e's}, 2 \text{ t's}, 2 \text{ n's}} \\&\frac{8!}{2! \cdot 2! \cdot 2!} = \frac{40320}{2 \cdot 2 \cdot 2} = 5040\end{aligned}$$

7. Factor: $n^3 + n^2 - n - 1$

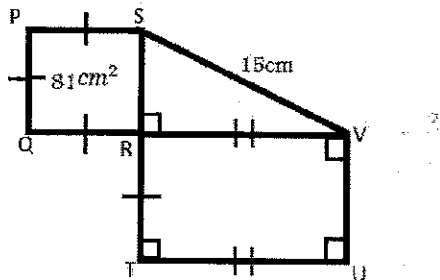
Solution:

$$\begin{aligned}n^2(n + 1) - 1(n + 1) \\(n^2 - 1)(n + 1) \\(n - 1)(n + 1)^2\end{aligned}$$



or $(n - 1)(n + 1)(n + 1)$

8. \square PQRS is a square whose area is 81 cm^2 . Find the area of \square RTUV



Solution:

The area of square PQRS is 81 cm^2 .

One side is equal to the square root of 81, which is 9 cm.

Sides SR and RT are equal to 9 cm.

To find Side RV:

$$9^2 + x^2 = 15^2$$

$$x^2 = 15^2 - 9^2$$

$$x^2 = 225 - 81$$

$$x^2 = 144$$

$$x = 12$$

$$RV = 12\text{ cm}$$

To find the area of rectangle RTUV:

$$RT * RV = 9 * 12$$

$$\text{area} = 108\text{ cm}^2$$

