

# Worcester County Mathematics League

Freshman Meet 2 – January 10, 2007

Round 1: Algebraic Word Problems

All answers must be in simplest exact form

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1. The sum of the numerator and denominator of a fraction is 8. If the numerator is doubled and the denominator is increased by 1, the value of the resulting fraction is 1. Find the fraction, reduced to lowest terms.
2. Four years ago, Bryan was half the age of Sterling. Now Sterling is 5 years older than Bryan. How old is Sterling now?
3. If the current speed limit on a 100 km stretch of highway is decreased by 10 km/hr, the trip will take 30 minutes longer than it would by following the current speed limit. What is the current speed limit in km/hr?

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_ years old

(3 pts.) 3. \_\_\_\_\_ km/hr

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Round 2: Number Theory

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

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1. Find the *least* three-digit number that is itself a perfect square but the sum of whose digits is not a perfect square.
  
  
  
  
  
  
  
  
  
  
2. Convert  $1324_5$  to base 7. (Note: the subscript indicates the number's base.)
  
  
  
  
  
  
  
  
  
  
3. Let  $A = 252$  and  $B = 378$ . Find the ratio of the greatest common factor of A and B to the least common multiple of A and B. Express your answer as a fraction reduced to lowest terms.

## ANSWERS

(1 pt.)      1. \_\_\_\_\_

(2 pts.)      2. \_\_\_\_\_

(3 pts.)      3. \_\_\_\_\_

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Round 3: Operations on Fractions, Decimals, Percents  
and Percentage Word Problems

All answers must be in simplest exact form

**NO CALCULATOR ALLOWED**

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1. Evaluate  $0.\bar{8} - 0.\bar{7} + \frac{2}{3} - 0.\bar{5} + 0.\bar{4} - \frac{1}{3} + 0.\bar{2} - 0.\bar{1}$ .

2. Two-thirds is  $\frac{3}{4}\%$  of what improper fraction?

3. Write the following as a reduced ratio of two integers:  $1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}}$

## ANSWERS

(1 pt.) 1. \_\_\_\_\_

(2 pts.) 2. \_\_\_\_\_

(3 pts.) 3. \_\_\_\_\_

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Round 4: Set Theory

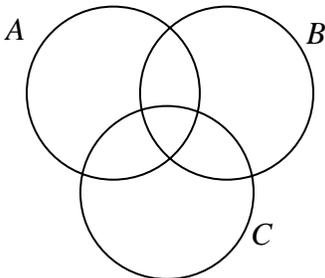
All answers must be in simplest exact form

NOTE:  $S'$  indicates the complement of the set  $S$

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- Using the Venn diagram in the answer section below, shade the region represented by the set  $(A \cap B) \cap C'$ .
- Let the universal set  $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ ,  $A = \{2, 3, 5, 7, 9\}$  and  $B = \{\text{the prime numbers less than } 10\}$ . How many subsets will the set  $(A \cap B)'$  have? (Note: For a set  $C$ , the empty set and  $C$  itself are considered subsets of  $C$ .)
- A high school principal reported that out of 150 members in the graduating class, 75 students studied French, 70 studied Latin and 60 studied Spanish. Also, 40 studied French and Latin, 25 studied Latin and Spanish, 30 studied French and Spanish, while 13 studied all three languages. How many graduates studied none of the three languages?

## ANSWERS

- (1 pt.) 1. 
- (2 pts.) 2. \_\_\_\_\_
- (3 pts.) 3. \_\_\_\_\_

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## TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

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1. When Mr. Rich died he left  $\frac{1}{2}$  of his estate to Mr. Sullivan,  $\frac{1}{6}$  of his estate to Mr. Yanco, and  $\frac{1}{8}$  to Mr. Bloom. The remaining \$150,000 was donated to WOCOMAL. How much did Mr. Bloom receive?
2. Let  $a$  and  $b$  be positive integers with  $a \neq b$ . If  $a + b$  is divisible by 3 and  $a - b$  is divisible by 5, what is the smallest possible value of the product  $ab$ ?
3. On a recent vocabulary quiz 6.25% of the class averaged a score of 96,  $16\frac{2}{3}\%$  of the class averaged an 84, 50% averaged a 70, and the rest of the class averaged a 48. What was the average score by the class on the quiz?
4. At a children's party, 13 had candy, 14 had gum and 20 had ice cream. Also, 6 had candy and gum, 10 had candy and ice cream, and 7 had gum and ice cream. Four had candy, gum and ice cream. How many children were at the party?
5. How many squares of any size are there on a standard 8 square by 8 square chessboard?
6. I am thinking of a two-digit number less than 50 such that the sum of the squares of the digits is 80 and the square of the sum of the digits is 144. Find the number.
7. If 2 apples weigh the same as 3 oranges and 3 oranges weigh the same as 4 bananas, how many bananas weigh the same as 5 apples?
8. Find the slope of the line through the point (6, 8) and the midpoint of the segment whose endpoints are (-7, 20) and (15, -10).

# Worcester County Mathematics League

All answers must be in simplest exact form!

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ANSWER SHEET – TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. \_\_\_\_\_

2. \_\_\_\_\_

3. \_\_\_\_\_

4. \_\_\_\_\_

5. \_\_\_\_\_

6. \_\_\_\_\_

7. \_\_\_\_\_

8. \_\_\_\_\_

# Worcester County Mathematics League

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## ANSWERS

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### Round 1

1.  $\frac{3}{5}$

2. 14

3. 50

### Round 2

1. 256

2. 424 or  $424_7$

3.  $\frac{1}{6}$

### Round 3

1.  $\frac{4}{9} = 0.\overline{4}$

2.  $\frac{800}{9}$

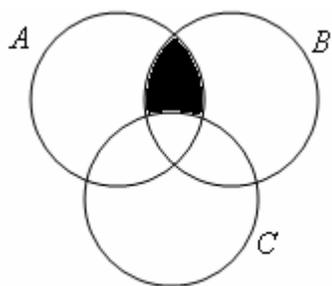
3.  $\frac{31}{18}$

### Round 4

1.  $\longrightarrow$

2. 64

3. 27



### Team Round

1. \$90,000

2. 14

3. 68

4. 28

5. 204

6. 48

7. 10

8.  $\frac{3}{2} = 1.5 = 1\frac{1}{2}$

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 BRIEF SOLUTIONS

Round 1

1. Let the numerator of the fraction be  $A$ . Then the denominator is  $8 - A$ . So,

$$\frac{2A}{9 - A} = 1 \Rightarrow 2A = 9 - A \Rightarrow A = 3 \text{ and } B = 5.$$

2. Let  $S =$  Sterling's age and  $B =$  Bryan's age. Then,  $S = B + 5$  and  $\frac{1}{2}(S - 4) = B - 4$ . This implies that

$$\frac{1}{2}(B + 1) = B - 4 \Rightarrow B = 9 \text{ and } S = 14.$$

3. Let  $r =$  the current speed limit in km/hr. Then,  $\frac{100}{r} + \frac{1}{2} = \frac{100}{r - 10} \Rightarrow 100(r - 10) + \frac{1}{2}r(r - 10) = 100r \Rightarrow$

$$\frac{1}{2}r^2 - 5r - 1000 = 0 \Rightarrow r^2 - 10r - 2000 = 0 \Rightarrow (r - 50)(r + 40) = 0 \Rightarrow r = 50.$$

Round 2

1. Carefully catalogue the perfect squares: the sum of the digits of 100 is 1, the sum of the digits of 121 is 4, the sum of the digits of 144 and 225 is 9, and the sum of the digits of 169 and 196 is 16. The sum of the digits of 256 is 13.

2. One way is to convert the base-5 number into base-10 and then convert this base-10 number into base-7:

$$1234_5 = 1 \cdot 125 + 3 \cdot 25 + 2 \cdot 5 + 4 = 125 + 75 + 10 + 4 = 214_{10} = 4 \cdot 49 + 2 \cdot 7 + 4 = 424_7.$$

3. First,  $252 = 2^2 \cdot 3^2 \cdot 7$  and  $378 = 2 \cdot 3^3 \cdot 7$ . So,  $\text{GCF}(252, 378) = 2 \cdot 3^2 \cdot 7$  and  $\text{LCM}(252, 378) = 2^2 \cdot 3^3 \cdot 7$ . Hence,

$$\frac{2 \cdot 3^2 \cdot 7}{2^2 \cdot 3^3 \cdot 7} = \frac{1}{6}.$$

Round 3

1. Recognize that all of the terms can be written as fractions whose denominators are 9:

$$0.\bar{8} - 0.\bar{7} + \frac{2}{3} - 0.\bar{5} + 0.\bar{4} - \frac{1}{3} + 0.\bar{2} - 0.\bar{1} = \frac{8}{9} - \frac{7}{9} + \frac{6}{9} - \frac{5}{9} + \frac{4}{9} - \frac{3}{9} + \frac{2}{9} - \frac{1}{9} = \frac{4}{9}.$$

2. Let  $x$  be the improper fraction. Then,  $\frac{\frac{2}{3}}{x} = \frac{\frac{3}{400}}{100} \Rightarrow \frac{3}{400}x = \frac{200}{3} \Rightarrow x = \frac{800}{9}$ .

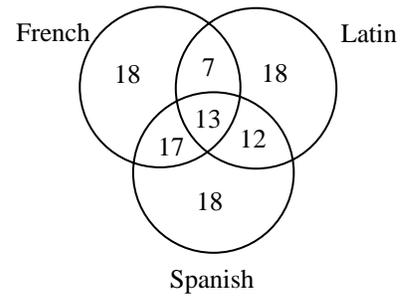
$$3. 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}} = 1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{1 + \frac{1}{3}}}} = 1 + \frac{1}{1 + \frac{1}{2 + \frac{3}{5}}} = 1 + \frac{1}{1 + \frac{5}{13}} = 1 + \frac{13}{18} = \frac{31}{18}.$$

Round 4

1. Carefully shade the intersection of A and B, then "cross-hatch" that region with the intersection of the complement of C.

2.  $A \cap B = \{2, 3, 5, 7\} \Rightarrow (A \cap B)' = \{1, 4, 6, 8, 9, 10\}$ . This set has  $2^6 = 64$  subsets.

3. One way: Use a carefully denoted Venn diagram. This accounts for 123 students, leaving 27 who do not take any language.



### Team Round

1. Mr. Rich gave  $\frac{12}{24} + \frac{4}{24} + \frac{3}{24} = \frac{19}{24}$  of his estate to Mr. Sullivan, Mr. Bloom and Mr. Yanco. Thus, he donated

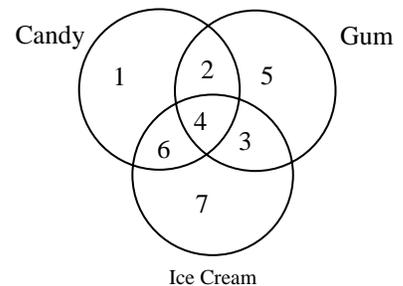
$\frac{5}{24}$  of it to WOCOMAL. Let  $x$  be the total value of his estate. Then,  $\frac{5}{24}x = \$150,000 \Rightarrow x = \$720,000$ . Mr.

Rich gave Mr. Bloom  $\frac{1}{8}(\$720,000) = \$90,000$ .

2. Since  $a$  and  $b$  are not equal, to make the product  $ab$  as small as possible,  $a - b = 5$ . Two numbers  $a$  and  $b$  which satisfy this equation and also satisfy  $a + b$  being a multiple of 3 are 7 and 2. Therefore, the smallest product is  $7 \cdot 2 = 14$

3. Of the students in the class,  $6.25\% = \frac{1}{16}$  scored a 96,  $16\frac{2}{3}\% = \frac{1}{6}$  scored an 84,  $50\% = \frac{1}{2}$  scored a 70 and the remaining  $27\frac{1}{12}\% = \frac{13}{48}$  scored a 48. So, the class average on the quiz was

$$\frac{1}{16}(96) + \frac{1}{6}(84) + \frac{1}{2}(70) + \frac{13}{48}(48) = 6 + 14 + 35 + 13 = 68.$$



4. Use a Venn diagram similar to #3 from Round 4:

There were 28 children at the party.

5. From the largest square to the smallest squares, there is one 8 by 8 square, four 7 by 7 squares, nine 6 by 6 squares, sixteen 5 by 5 squares, twenty-five 4 by 4 squares, thirty-six 3 by 3 squares, forty-nine 2 by 2 squares, and sixty-four 1 by 1 squares. Hence, there are a total of  $1 + 4 + 9 + 16 + 25 + 36 + 49 + 64 = 204$  squares.

6. Let the number have the digits  $A$  and  $B$ . Then,  $(A + B)^2 = 144 \Rightarrow A + B = 12$ . Since the number is less than 50, it must be either 48 or 39. However,  $4^2 + 8^2 = 80 \neq 3^2 + 9^2$ , so I am thinking of the number 48.

7. If 2 apples = 3 oranges, and 3 oranges = 4 bananas, then 2 apples = 4 bananas. Then, 5 apples = 10 bananas.

8. The midpoint of the line segment whose endpoints are  $(-7, 20)$  and  $(15, -20)$  is  $\left(\frac{15+(-7)}{2}, \frac{-10+20}{2}\right) = (4, 5)$ . Then,

the slope of the line containing  $(6, 8)$  and  $(4, 5)$  is  $\frac{8-5}{6-4} = \frac{3}{2}$ .