WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015
Round 1: Algebraic Word Problems

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. A salesman earns a commission of \$2100 after making a sale of \$35,000. At the same commission rate, how much would he earn after making a sale of \$46,000?

2. Jan's bathtub fills up in 8 minutes when its drain is plugged. When the drain is unplugged, the full tub empties out in 15 minutes. How many minutes will it take Jan's tub to fill up if she turns on the faucet but forgets to plug the drain?

3. Alex and Laura each own some gold, totaling 12 oz. between them. Alex says to Laura, "If I give you one third of my gold, then you give me one half of your new total of gold, and finally I give you one third of my new total of gold, we will have equal amounts of gold." What was Alex's original amount of gold?

ANSWERS

(1 pt.) 1. \$________________________________

(2 pts.) 2. ________________________________ minutes

(3 pts.) 3. ________________________________ ounces

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015
Round 2: Number Theory

All answers must be in simplest exact form in the answer section
NO CALCULATOR ALLOWED

1. What is the largest prime factor of 6630?

2. Find the product of the least common multiple and greatest common factor of the numerator and denominator of the simplified fraction form of $0.2\overline{7}$.

3. Among the integers 1 to 300, how many are not divisible by 7 that are divisible by either 3 or 5?

ANSWERS

(1 pt.) 1. ______________________________

(2 pts.) 2. ______________________________

(3 pts.) 3. ______________________________

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015
Round 3: Operations on Numerical Fractions, Decimals, Percents, and Percentage Word Problems

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Simplify: $19+9(.\bar{2}+.\bar{3})^2$

2. In her first 8 games, a basketball player successfully made 40% of the shots she attempted. If in the next game she makes 10 out of 12 shots, her season average will rise to 45%. How many shots did the player attempt in the first 8 games?

3. Simplify: $\frac{(6)^3-(6)^2}{(6)^2-1}\times\frac{(5)^3-(5)^2}{(5)^2-1}\times\cdots\times\frac{(2)^3-(2)^2}{(2)^2-1}$

ANSWERS

(1 pt.) 1. ____________________

(2 pts.) 2. ____________________ shots attempted

(3 pts.) 3. ____________________

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015
Round 4: Set Theory

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. A survey of college students finds that 30% like country music, 40% like rock, and 10% like both country and rock. What percentage of students like neither type of music?

2. Set A = {1, 2, 3, 4, 5, 6} and set B = {1, 3, 5, 7, 9, 11}. How many subsets of A are NOT subsets of B?

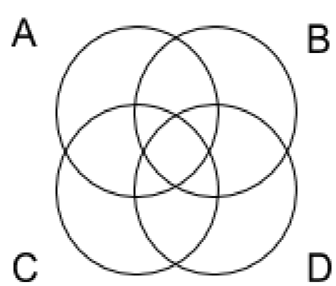
3. For two sets A and B contained in a universal set U, define the following set operation $A \diamond B = (A \cup B) \cap (A \cap B)^C$ where A^C denotes the complement of A. Let A, B, C, and D be sets contained in a universal set S. On the figure provided, shade: $[(A \diamond B) \diamond C] \diamond D$

ANSWERS

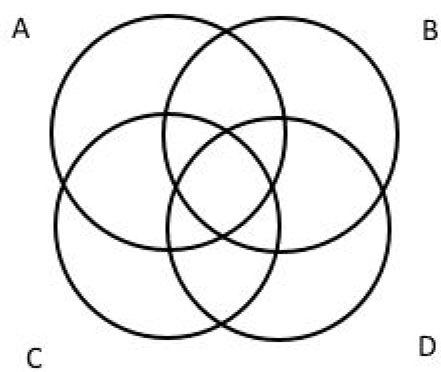
(1 pt.) 1. ____________________________ %

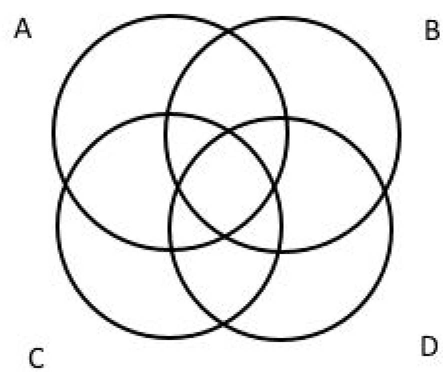
(2 pts.) 2. ____________________________ subsets

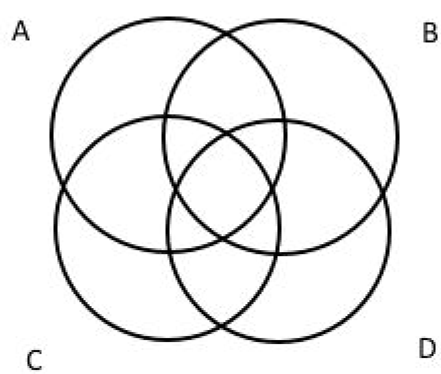
(3 pts.) 3.

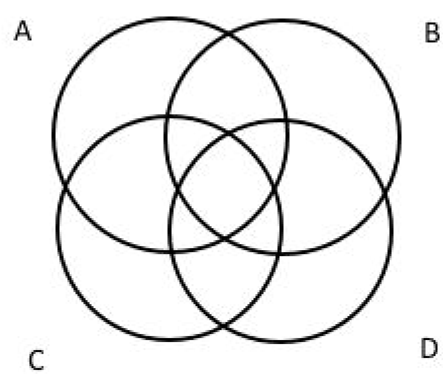


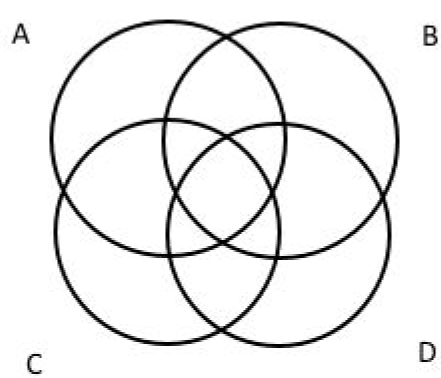
Freshman Meet 2 - December 16th, 2015
Round 4: Question 3 SCRAP PAPER

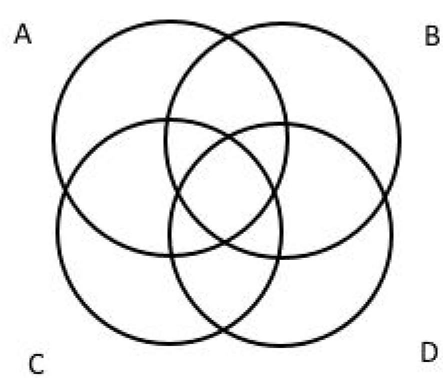












WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015

Team Round

All answers must either be in simplest exact form or rounded to EXACTLY three decimal places, unless stated otherwise. (3 points each)

APPROVED CALCULATORS ALLOWED

1. A rectangular pool is 7 meters longer than it is wide. It is surrounded by a concrete sidewalk which is 1.5 meters wide and the area of the sidewalk is 120 sq. meters. Find the length of the pool.

2. $2010221_3 + 2211201_3 = x_9$. Find x.

3. If Bob was 24 when his son Jim was born and now he is 3 times as old as Jim, how many years from now will Jim's age be 60% of Bob's age?

4. Let S be the grand set and let A be a subset of S. If A^C denotes the complement of A, then simplify: $\{A \cap [A \cup (A \cap (A \cup A^C)^C)^C]^C\}^C$

5. Let A be the sum of fifty consecutive integers. Express the sum of the next fifty consecutive integers in terms of A.

6. Suppose that $212.12_3 = x_6$. Find x.

7. Lucy and Paula, at their respective constant rates, ran a 480m race twice. In the first race, Lucy let Paula start 48m in front of the starting line, but Lucy won the race by one tenth of a minute. In the second race, Lucy let Paula start 144m ahead, and Paula won by 2 seconds. In meters per second, what is Paula's constant running speed?

8. Sal signed an employment contract which stipulates that in week one, he will earn x dollars. Every following week he will earn $12.50 more than he did in the week previous. If Sal earned $78,975 after working for 52 weeks, determine x.

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015
Team Round Answer Sheet

1. ____________________________ meters

2. ____________________________

3. ____________________________ years

4. ____________________________

5. ____________________________

6. ____________________________

7. ____________________________ meters/second

8. $___________________________

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015 ANSWER KEY

Round 1:

1. 2760 (South)
2. $17\frac{1}{7}$ or $\frac{120}{7}$ (Assabet Valley)
3. 9 (Worcester Academy)

Round 2:

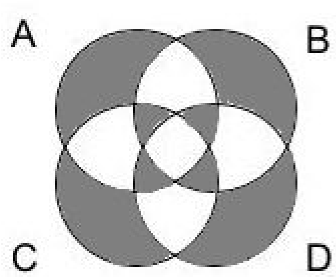
1. 17 (Notre Dame Academy)
2. 90 (Bromfield)
3. 120 (Algonquin)

Round 3:

1. $21\frac{7}{9}$ or $\frac{196}{9}$ or $21.\overline{7}$ (Bartlett)
2. 92 (Burncoat)
3. $\frac{1440}{7}$ or $205\frac{5}{7}$ (QSC)

Round 4:

1. 40 (Shrewsbury)
2. 56 (Worcester Academy)
3. (AMSA)



TEAM Round

1. 22 (Notre Dame Academy)
2. 4878 (Assabet Valley)
3. 24 (Quaboag)
4. S (Bromfield)
5. A + 2500 or 2500 + A (West Boylston)
6. 35.32 (Tantasqua)
7. 12 (West Boylston)
8. 1200 (Bromfield)

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 - December 16th, 2015 - SOLUTIONS

Round 1: Algebraic Word Problems

1. A salesman earns a commission of $2100 after making a sale of $35,000. At the same commission rate, how much would he earn after making a sale of $46,000?

Solution 1: To determine the salesman's rate of commission, we divide $\frac{\$2100}{\$35000} = \frac{\$21}{\$350} = \frac{3}{50} = \frac{6}{100}$. Then, to determine the commission that the salesman will earn on the new sale, we multiply $\$46000 \times \frac{6}{100} = \$460 \times 6 = \$2760.$

Solution 2: Let $\$100 \times x$ be the commission the salesman will earn on the second sale. Then it must be true that

$$\frac{2100}{35000} = \frac{100 \times x}{46000}$$
$$\frac{21}{350} = \frac{x}{460}$$
$$x = \frac{46 \times 3}{5} = \frac{126}{5}.$$

Since we have that the commission equals $\$100 \times x$, then the commission is:

$$\$100 \times x = \$100 \times \frac{126}{5}$$
$$\$100 \times x = \$20 \times 126 = \$2760.$$

2. Jan's bathtub fills up in 8 minutes when its drain is plugged. When the drain is unplugged, the full tub empties out in 15 minutes. How many minutes will it take Jan's tub to fill up if she turns on the faucet but forgets to plug the drain?

Solution 1: The faucet has a rate of filling of 1 tub/8 min and the drain has an effective rate of filling of -1 tub/15 min. If we set t to be equal to the time necessary to fill the tub when the faucet is on and the drain is not plugged, then we have that

$$\left(\frac{1\text{ tub}}{8\text{ min}} - \frac{1\text{ tub}}{15\text{ min}}\right) \times t = 1\text{ tub}$$
$$\frac{15\text{ tubs}}{\text{min}}t - \frac{8\text{ tubs}}{\text{min}}t = 120\text{ tubs}$$

$$\frac{7\text{ tubs}}{\text{min}}t = 120\text{ tubs}$$
$$t = 120\text{ tubs} \times \frac{1\text{min}}{7\text{ tubs}}$$
$$t = \frac{120}{7}\text{ min} = 17\frac{1}{7}\text{ minutes}$$

Solution 2: We have that the faucet fills one tub in 8 minutes and that the drain can empty one tub in 15 minutes. We can express these rates alternatively as the faucet fills 15 tubs in 120 minutes and the drain empties 8 tubs in 120 minutes (note that 120 is the least common multiple of 8 and 15). Therefore, if the faucet is running and the drain is not plugged, 7 tubs will be filled in 120 minutes. Hence, it takes precisely $\frac{120}{7}$ minutes to fill one tub.

3. Alex and Laura each own some gold, totaling 12 oz. between them. Alex says to Laura, "If I give you one third of my gold, then you give me one half of your new total of gold, and finally I give you one third of my new total of gold, we will have equal amounts of gold." What was Alex's original amount of gold?

Solution 1(Solve Completely): Let x denote Alex's original amount of gold. Therefore, we know that Laura begins with 12 - x ounces of gold. Let's draw a table showing the amounts of gold each person has at each step of the proposed plan:

	ALEX	LAURA
Initial	x	$12 - x$
Step 1	$x - \frac{1}{3}x = \frac{2}{3}x$	$12 - x + \frac{1}{3}x = 12 - \frac{2}{3}x$
Step 2	$\frac{2}{3}x + \frac{1}{2}(12 - \frac{2}{3}x) = \frac{1}{3}x + 6$	$12 - \frac{2}{3}x - \frac{1}{2}(12 - \frac{2}{3}x) = 6 - \frac{1}{3}x$
Step 3	$\frac{1}{3}x + 6 - \frac{1}{3}(\frac{1}{3}x + 6) = \frac{2}{9}x + 4$	$6 - \frac{1}{3}x + \frac{1}{3}(\frac{1}{3}x + 6) = 8 - \frac{2}{9}x$

Now we know they end with equal amounts of gold, so we have that

$$\frac{2}{9}x + 4 = 8 - \frac{2}{9}x$$
$$\frac{4}{9}x = 4$$
$$x = 9 \times \frac{4}{4} = 9.$$

Solution 2 (Use Fact about Total Gold): Follow the same steps as in Solution 1 but note that because there is always a total of 12 ounces between Alex and Laura, we only need to find an expression for one person's amount after each step and then we know that the other person must have 12 minus that amount:

	ALEX	LAURA
Initial	x	$12-x$
Step 1	$x-\frac{1}{3}x=\frac{2}{3}x$	$12-\frac{2}{3}x$
Step 2	$12-(6-\frac{1}{3}x)=6+\frac{1}{3}x$	$12-\frac{2}{3}x-\frac{1}{2}(12-\frac{2}{3}x)=6-\frac{1}{3}x$
Step 3	$\frac{1}{3}x+6-\frac{1}{3}(\frac{1}{3}x+6)=\frac{2}{9}x+4$	

Now we know they end with equal amounts of gold, and there is a total of 12 ounces, so we have that Alex must end with 6 ounces. Therefore

$$\frac{2}{9}x+4=6$$
$$\frac{2}{9}x=2$$
$$x=9.$$

Solution 3 (Work Backwards - Use Inverses): Since we are told that Alex and Laura end with equal amounts of gold and there is a total of 12 ounces, we know that after the final step each person must have 6 ounces. Now examine the trading process backwards and perform the inverse of each step to reach the desired answer.

In the final step (step 3), Alex gives Laura one third of his gold, which in other words means he multiplies his own total of gold by $\frac{2}{3}$. Therefore, the inverse of this step is simply to multiply his final total of gold by $\frac{3}{2}$. Since $\frac{3}{2}\times 6=9$, Alex must have 9 ounces before step 3, and so Laura must have 3 ounces before step 3.

In step 2, Laura gives half of her gold to Alex (or alternatively multiplies her own total by $\frac{1}{2}$) and she ends this step with 3 ounces. The inverse of this step is simply to multiply her resulting total of gold of 3 ounces by $\frac{2}{1}$, which means before step 2 Laura has 6 ounces and so Alex must have have 6 ounces as well.

In step 1, Alex gives Laura one third of his gold, which in other words means he multiplies his own total of gold by $\frac{2}{3}$. Therefore, the inverse of this step is simply to multiply his resulting total of gold by $\frac{3}{2}$, which means Alex has 9 ounces before step 1. Notice that this is the desired answer.

This reasoning can be visualized in the following table:

	ALEX	LAURA
Final Gold:	6	6
Gold before Step 3:	$\frac{3}{2}\times 6=9$	$12-9=3$
Gold before Step 2:	$12-6=6$	$\frac{2}{1}\times 3=6$
Gold before Step 1:	$\frac{3}{2}\times 6=9$	

Solution 4 (Guess and Check): We can guess logical amounts that Alex might begin with and then check to see if the each person ends with 6 ounces as desired.

Guess 1: Alex starts with 6 ounces

	ALEX	LAURA
Initial Gold:	6	$= 12 - 6 = 6$
Gold after Step 1:	$\frac{2}{3} \times 6 = 4$	$= 12 - 4 = 8$
Gold after Step 2:	$= 12 - 4 = 8$	$\frac{1}{2} \times 8 = 4$
Gold after Step 3:	$\frac{2}{3} \times 8 = \frac{16}{3} \neq 6$	

Since Guess 1 left Alex with not enough gold at the end, try:

Guess 2: Alex starts with 9 ounces

	ALEX	LAURA
Initial Gold:	9	$= 12 - 9 = 3$
Gold after Step 1:	$\frac{2}{3} \times 9 = 6$	$= 12 - 6 = 6$
Gold after Step 2:	$= 12 - 3 = 9$	$\frac{1}{2} \times 6 = 3$
Gold after Step 3:	$\frac{2}{3} \times 9 = 6$	$= 12 - 6 = 6$

This leaves both people with 6 ounces as desired, so 9 ounces must be Alex's initial gold.

Round 2: Number Theory

1. What is the largest prime factor of 6630?

Solution: Start by dividing by the smallest prime numbers and working up: $6630 = 2 \cdot 3315 = 2 \cdot 3 \cdot 1105 = 2 \cdot 3 \cdot 5 \cdot 221.$

Now 221 is not divisible by 7, since 7 divides 210 evenly but not the remaining 11. Likewise 221 is not divisible by 11, since 11 divides 220 evenly but leaves 1 remaining.

We can divide 221 by 13, however, leaving a quotient of 17. Since 17 is prime, we conclude that the largest prime factor of 6630 is 17.

2. Find the product of the least common multiple and greatest common factor of the numerator and denominator of the simplified fraction form of $0.2\overline{7}$.

Solution: To determine the fraction form of $0.2\overline{7}$, let $x = 0.2\overline{7}$. We have

$$x = 0.2\overline{7}$$
$$100x - 10x = 27.\overline{7} - 2.\overline{7}$$
$$90x = 25$$
$$x = \frac{25}{90} = \frac{5}{18}.$$

Now 5 and 18 share no common prime factors, so gcf(5, 18) = 1 and lcm(5, 18) = $5 \times 18 = 90$. Therefore, the desired answer is 90.

3. Among the integers 1 to 300, how many are not divisible by 7 that are divisible by either 3 or 5?

Solution 1 (Simple Counting):

Since 300/3 = 100, there are 100 integers divisible by 3.
Since 300/5 = 60, there are 60 integers divisible by 5.

But there are not 100 + 60 integers divisible by either 3 <u>or</u> 5, since we would be double counting integers which are divisible by both 3 <u>and</u> 5.

Since $300/(3 \times 5)$ = 20, there are 20 integers divisible by 3 <u>and</u> 5. Therefore, there are 100 + 60 - 20 = 140 integers divisible by 3 or 5 between 1 and 300.

To complete the problem, we need to eliminate from these 140 integers any integer which is a multiple of 7.
Since $300/(3 \times 7) = 14$ + remainder, there are 14 integers divisible by both 3 <u>and</u> 7.
Since $300/(5 \times 7) = 8$ + remainder, there are 8 integers divisible by both 5 <u>and</u> 7.

Notice again that we are double counting integers here which are divisible by 3 <u>and</u> 5 <u>and</u> 7.

Since $300/(3 \times 5 \times 7) = 300/105 = 2$ + remainder, there are 2 integers which divisible by 3 <u>and</u> 5 <u>and</u> 7.

To conclude: (100 + 60 - 20) = 140 gives the number of integers divisible by 3 or 5.

(14 + 8 - 2) = 20 gives the number of integers divisible by 3 or 5 which are also divisible by 7

So 140 - 20 = 120 the number of integers divisible by 3 or 5 but not by 7.

Solution 2 (Inclusion-Exclusion): To solve this problem, we can use the inclusion-exclusion principle.

Let A be the set of integers between 1 and 300 which are divisible by 3 but not by 7.
Let B be the set of integers between 1 and 300 which are divisible by 5 but not by 7.

In this problem, we want to determine the number of elements in the union of A and B, and we express this number in set notation as $|A \cup B|$.

The inclusion-exclusion principle states that $|A \cup B| = |A| + |B| - |A \cap B|$. That is, we add the number of integers divisible by 3 but not 7 to the number of integers divisible by 5 but not 7 and then subtract the number of integers divisible by both 3 and 5 but not 7.

Since 300/3 = 100, there are 100 integers divisible by 3.
Since $300/(3 \times 7) = 14$ + remainder, there are 14 integers divisible by both 3 and 7.
Therefore, $|A|$ = 100 - 14 = 86.

Since 300/5 = 60, there are 60 integers divisible by 5.
Since $300/(5 \times 7) = 8$ + remainder, there are 8 integers divisible by both 5 and 7.
Therefore, $|B|$ = 60 - 8 = 52.

Since $300/(3 \times 5)$ = 20, there are 20 integers divisible by 3 and 5.
Since $300/(3 \times 5 \times 7) = 300/105 = 2$ + remainder, there are 2 integers divisible by 3, 5 and 7.
Therefore, $|A \cap B|$ = 20 - 2 = 18.

In conclusion, $|A \cup B| = |A| + |B| - |A \cap B|$ = 86 + 52 - 18 = 120.

Round 3: Operations on Numerical Fractions, Decimals, Percents, and Percentage Word Problems

1. Simplify: $19 + 9(.\bar{2} + .\bar{3})^2$

Solution: First we note that $.\bar{2} + .\bar{3} = 0.\bar{5}$. Now set $x = 0.\bar{5}$. We have that

$$10x - x = 5.\bar{5} - 0.\bar{5} = 5$$
$$9x = 5$$
$$x = \frac{5}{9}.$$

We can now plug this value of x into the original expression to get

$$19 + 9 \times \left(\frac{5}{9}\right)^2 =$$
$$19 + \frac{25}{9} =$$
$$19 + 2\frac{7}{9} = 21\frac{7}{9}.$$

2. In her first 8 games, a basketball player successfully made 40% of the shots she attempted. If in the next game she makes 10 out of 12 shots, her season average will rise to 45%. How many shots did the player attempt in the first 8 games?

Solution: Let x be the number of shots the player attempted in the first 8 games. Since her average through 8 games is 40%, we know that the player has successfully made .4x shots in the first 8 games. Therefore, from the given information about her next game, we have that

$$\frac{0.4x+10}{x+12}=0.45$$
$$0.4x+10=0.45(x+12)$$
$$0.4x+10=0.45x+5.4$$
$$4.6=0.05x$$
$$x=92.$$

3. Simplify: $\frac{(6)^3-(6)^2}{(6)^2-1}\times\frac{(5)^3-(5)^2}{(5)^2-1}\times\cdots\times\frac{(2)^3-(2)^2}{(2)^2-1}$

Solution 1 (Solve Directly): Evaluate each factor in the product on its own:

$$\frac{6^3-6^2}{6^2-1}\times\frac{5^3-5^2}{5^2-1}\times\frac{4^3-4^2}{4^2-1}\times\frac{3^3-3^2}{3^2-1}\times\frac{2^3-2^2}{2^2-1}=$$
$$\frac{216-36}{36-1}\times\frac{125-25}{25-1}\times\frac{64-16}{16-1}\times\frac{27-9}{9-1}\times\frac{8-4}{4-1}=$$
$$\frac{180}{35}\times\frac{100}{24}\times\frac{48}{15}\times\frac{18}{8}\times\frac{4}{3}=$$
$$\frac{36}{7}\times\frac{25}{6}\times\frac{16}{5}\times\frac{9}{4}\times\frac{4}{3}=$$
$$\frac{6}{7}\times\frac{5}{1}\times\frac{4}{1}\times\frac{3}{1}\times\frac{4}{1}=$$
$$\frac{20\times72}{7}=\frac{2\times720}{7}=\frac{1440}{7}$$

Solution 2 (Simplify general term first): Instead of evaluating each factor in the product on its own, attempt to simplify the expression first. Notice that each factor is of the form $\frac{n^3-n^2}{n^2-1}$. This can be simplified as:

$$\frac{n^3-n^2}{n^2-1}=$$
$$\frac{n^2(n-1)}{(n+1)(n-1)}=$$
$$\frac{n^2}{n+1}.$$

Now we can quickly rewrite each factor in the product:

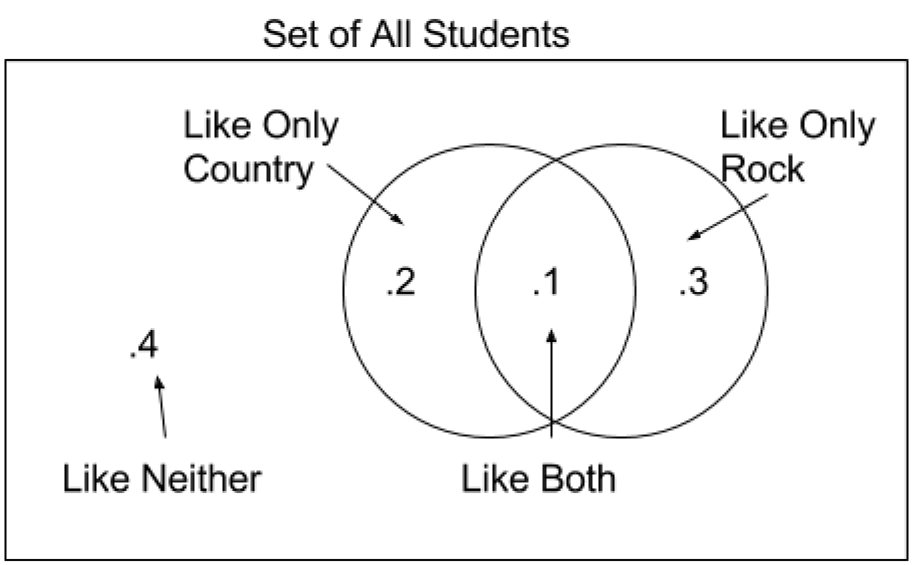
$$\frac{6^2}{7}\times\frac{5^2}{6}\times\frac{4^2}{5}\times\frac{3^2}{4}\times\frac{2^2}{3}=$$
$$\frac{6}{7}\times\frac{5}{1}\times\frac{4}{1}\times\frac{3}{1}\times\frac{2^2}{1}=$$
$$\frac{2}{7}\times 6!=$$
$$\frac{2}{7}\times 720=\frac{1440}{7}=205\frac{5}{7}.$$

Round 4: Set Theory

1. A survey of college students finds that 30% like country music, 40% like rock, and 10% like both country and rock. What percentage of students like neither type of music?

Solution 1 (Analytical): If 10% of students like both country and rock, then that implies that 20% only like country and 30% only like rock. Since the total number of students must add up to 100%, there must be 40% of students who like neither type of music.

Solution 2 (Venn Diagram): Set up a Venn diagram to represent the situation:



2. Set A = {1, 2, 3, 4, 5, 6} and set B = {1, 3, 5, 7, 9, 11}. How many subsets of A are NOT subsets of B?

Solution: Since sets A and B each have 6 elements, we know that each set has $2^6 = 64$ subsets. Further, note that $A \cap B = \{1,3,5\}$.

Let X be a subset of set A. X is a subset of set B if and only if X is a subset of $A \cap B$. Since $A \cap B$ has 3 elements, we know that it has $2^3 = 8$ subsets. So, of the 64 subsets of A, 8 of them are subsets of $A \cap B$ and are therefore subsets of B. That means there are 64 - 8 = 56 subsets of A that are not subsets of B.

3. For two sets A and B contained in a universal set U, define the following set operation $A \diamond B = (A \cup B) \cap (A \cap B)^C$ where A^C denotes the complement of A. Let A, B, C, and D be sets contained in a universal set S. On the figure provided, shade: $[(A \diamond B) \diamond C] \diamond D$

Solution 1 (Step-by-step): Begin by noting that the operation defined in the problem is precisely the symmetric difference between the sets A and B. In other words, $A \diamond B$ is equal to the set of all points that are contained in either A or B, but <u>not both A and B</u>.

Start by diagramming $A \diamond B$, which will include all points that contained in either A or B, but not both. This area is shaded below in Figure 1 with horizontal lines.

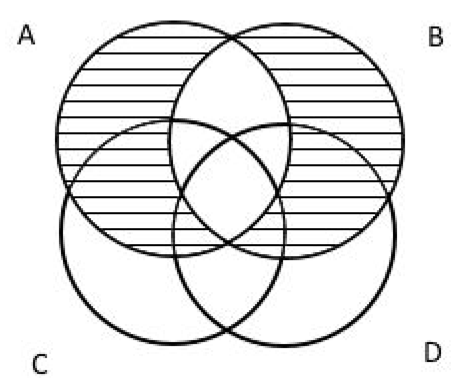


Fig. 1

Define the set $P \equiv A \diamond B$. So now we are interested in shading the area $P \diamond C$, which consists of all points found in either P or C, but not both. This area is shaded below in Figure 2 using diagonal lines.

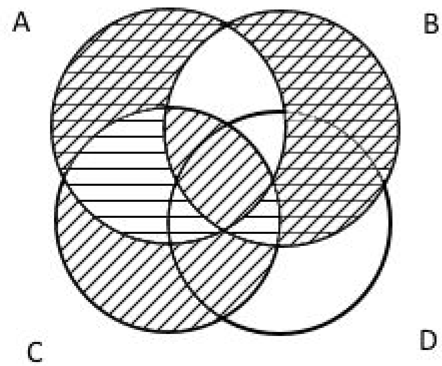


Fig. 2

Notice that the areas in Figure 2 that are only shaded with horizontal lines are the areas which appear in both P and in C, and are therefore excluded from $P \diamond C$. For clarity, we can now examine $P \diamond C$ in Figure 3 where all horizontal lines have been removed:

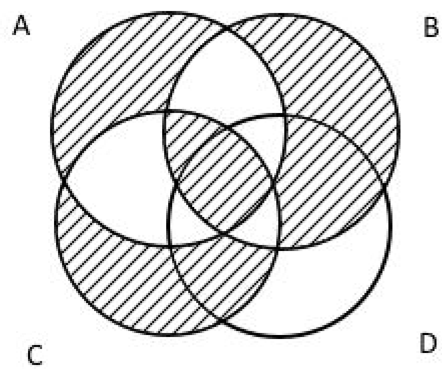


Fig. 3

Now define $Q \equiv P \diamond C$. To complete the problem, we need only to shade in $Q \diamond D$, which consists of all points contained in either Q or D but not both. This area is shaded in Figure 4 using solid black.

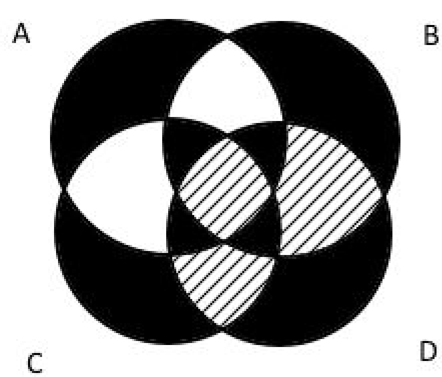


Fig. 4

Notice that the areas in Figure 4 shaded only with diagonal lines are areas contained in both Q and in D, and are therefore excluded from $Q \diamond D$. Hence, the final answer is given by Figure 5:

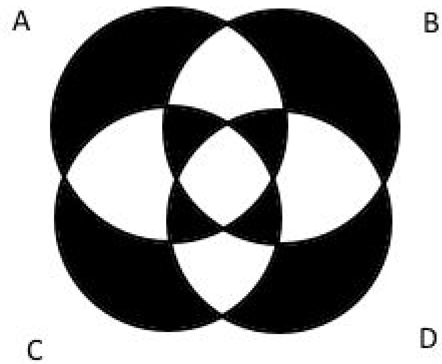


Fig. 5

Solution 2 (Exploit Symmetric Difference): The symmetric difference operator has the general property that when symmetric differences are taken over multiple sets, any regions which appear in an odd number of sets will be part of the final set. Therefore, we need only shade the regions in the venn diagram which appear either in one and only one set or in three and only three sets, giving us the answer immediately.

Team Round

1. A rectangular pool is 7 meters longer than it is wide. It is surrounded by a concrete sidewalk which is 1.5 meters wide and the area of the sidewalk is 120 sq. meters. Find the length of the pool.

Solution: Let x be the width of the pool. Therefore, we know that the area of pool is given by $x(x+7)$. Now since we know that the sidewalk is 1.5 meters wide, we know that the total length of the pool and sidewalk is $x+3$ and the total width of the sidewalk and pool is $x+7+3=x+10$. Since we are given that the area of the sidewalk is 120 sq. meters, it must be the case that

$$\begin{aligned}
&(x+3)(x+10)-x(x+7)=120\\
&x^2+3x+10x+30-x^2-7x=120\\
&x^2+3x+10x+30-x^2-7x=120\\
&6x+30=120\\
&6x=90\\
&x=15.
\end{aligned}$$

Finally, the question asks for the length of the pool, which is x + 7 = 22 meters.

2. $2010221_3 + 2211201_3 = x_9$. Find x.

Solution 1 (Convert to Base 10): Begin by adding the two base 3 numbers:

```
  2010221
+ 2211201
 11222122
```

Now convert this base 3 number into base 10:

11222122 = $(1 \cdot 3^7) + (1 \cdot 3^6) + (2 \cdot 3^5) + (2 \cdot 3^4) + (2 \cdot 3^3) + (1 \cdot 3^2) + (2 \cdot 3^1) + (2 \cdot 3^0) = 3635$

Now convert the base 10 number into base 9. Start by noting that $9^4 = 6561 > 3635$, so we know that this number is less than 5 digits in base 9.

$9^3 = 729$ and $\frac{3635}{729} = 4$ with a remainder of 719.
$9^2 = 81$ and $\frac{719}{81} = 8$ with a remainder of 71.
$9^1 = 9$ and $\frac{71}{9} = 7$ with a remainder of 8.
$9^0 = 1$ and $\frac{8}{1} = 8$.

Therefore, the desired answer is x = 4878.

Solution 2 (Direct Conversion): Since 9 is the square of 3, we can exploit the following trick to quickly convert each base 3 number into base 9 directly. Starting from the ones digit and working left, break down each base 3 number into pairs and then convert each pair into base 10 as follows:

2010221_3	2211201_3
2_3 01_3 02_3 21_3	2_3 21_3 12_3 01_3
2 1 2 7	2 7 5 1
2127	2751

Then we simply add the two resulting numbers in base 9 to achieve the desired result:

$2127_9 + 2751_9 = 4878_9$

And therefore x = 4878.

3. If Bob was 24 when his son Jim was born and now he is 3 times as old as Jim, how many years from now will Jim's age be 60% of Bob's age?

Solution: Let x be Jim's age now. The first statement tells us that Bob's age now must be x + 24. Moreover, we have that Bob is now 3 times as old as Jim, and so

$$24+x=3x$$
$$2x=24$$
$$x=12.$$

This means Jim is currently 12 and Bob is 36. Now let y be the number of years from now that Jim's age is 60% of Bob's age. We can rewrite the last part of the question as

$$12+y=0.6(36+y)$$
$$120+10y=6(36+y)$$
$$120+10y=216+6y$$
$$4y=96$$
$$y=24.$$

4. Let S be the universal set and let A be a subset of S. If A^C denotes the complement of A, then simplify: $\{A\cap[A\cup(A\cap(A\cup A^C)^C)^C]^C\}^C$

Solution: If we start from the middle we can immediately see that $A\cup A^C=S.$ We use this fact to simplify the remainder of the expression, working from the innermost set of parentheses outwards (note that the whitespace in the following is added for ease of viewing):

$$\{A\cap[A\cup(A\cap(S)^C)^C]^C\}^C$$
$$\{A\cap[A\cup(A\cap \varnothing\ )^C]^C\}^C$$
$$\{A\cap[A\cup(\quad \varnothing \quad)^C]^C\}^C$$
$$\{A\cap[A\cup \quad S \quad]^C\}^C$$
$$\{A\cap[\quad S \quad]^C\}^C$$
$$\{A\cap \quad \varnothing \quad\}^C$$
$$\{\quad \varnothing \quad\}^C$$
$$S$$

5. Let A be the sum of fifty consecutive integers. Express the sum of the next fifty consecutive integers in terms of A.

Solution: Define x such that $x+(x+1)+(x+2)+...+(x+49)=A$. Notice that if we sum up the next 50 integers $(x+50)+(x+51)\ +(x+52)+...+(x+99)$, each term of the second sum is precisely 50 larger than each respective term found in the sum in the first line. Since there are 50 terms, we have that the sum of the 50 integers after A can be written

$$[x+50]+[(x+1)+50]+[(x+2)+50]+...+[(x+49)+50]=$$
$$50(50)+x+(x+1)+...+(x+49)=$$
$$2500+A.$$

6. Suppose that $212.12_3=x_6$. Find x.

Solution: First convert the digits in front of the decimal point.

$$212_3=2(9)+1(3)+2(1)=23_{10}$$
$$23_{10}=18+5=3(6)+5(1)=35_6$$

Now let's examine the digits beyond the decimal point:

$$0.12_3=1(3^{-1})+2(3^{-2})$$
$$0.12_3=1\left(\frac{1}{3}\right)+2\left(\frac{1}{9}\right)=\frac{5}{9}_{10}$$
$$\frac{5}{9}_{10}=\frac{20}{36}_{10}=\frac{18}{36}+\frac{2}{36}=3\left(\frac{1}{6}\right)+2\left(\frac{1}{36}\right)=$$
$$3(6^{-1})+2(6^{-2})=\ .32_6$$

Therefore, we have that x = 35.32.

7. Lucy and Paula, at their respective constant rates, ran a 480m race twice. In the first race, Lucy let Paula start 48m in front of the starting line, but Lucy won the race by one tenth of a minute. In the second race, Lucy let Paula start 144m ahead, and Paula won by 2 seconds. In meters per second, what is Paula's constant running speed?

Solution 1: Let x be Paula's constant running speed. Since Lucy runs both races from the starting line at the same speed, we know that Lucy will complete race 1 and race 2 with the exact same finishing time. We use this fact to rewrite the problem as

$$\frac{480-48}{x}-6=\text{ Lucy's Time}=\frac{480-144}{x}+2$$
$$\frac{432}{x}=\frac{336}{x}+8$$
$$\frac{96}{x}=8$$
$$x=12.$$

Solution 2: Begin by noting that Lucy runs both races starting at the starting line and that she runs at a constant speed. This implies Lucy's time in both races is the same. Let t be Lucy's running time, R_P be Paula's speed in m/s, and R_L be Lucy's speed in m/s. From the statement of the problem, we can set up the following table:

		Rate	×	Time	=	Distance
RACE 1						
	Lucy	R_L		t		480
	Paula	R_P		t + 6		432
RACE 2						
	Lucy	R_L		t		480
	Paula	R_P		t - 2		336

From the the Race 2 equation for Paula, we solve for t:

$$R_P(t-2)=336$$
$$t-2=\frac{336}{R_P}$$
$$t=\frac{336}{R_P}+2$$

Now plug in this expression for t into the Race 1 equation for Paula:

$$R_P(t+6)=432$$
$$R_P(\frac{336}{R_P}+2+6)=432$$
$$336+8R_P=432$$
$$8R_P=96$$
$$R_P=12$$

8. Sal signed an employment contract which stipulates that in week one, he will earn x dollars. Every following week he will earn $12.50 more than he did in the week previous. If Sal earned $78,975 after working for 52 weeks, determine x.

Solution: Let x be Sal's earnings in week 1. We have that over the course of 52 weeks, Sal will earn

$$x + (x + 12.50(1)) + (x + 12.50(2)) + \ldots + (x + 12.50(51)) =$$
$$52x + 12.50(1 + 2 + \ldots + 51) =$$
$$52x + 12.50(\frac{51 \times 52}{2}) =$$
$$52x + 16575.$$

Therefore, to solve for x we have

$$52x + 16575 = 78975$$
$$52x = 62400$$
$$x = 1200.$$