



Freshman Meet 1 – October 30, 2013
Round 1: Evaluation of Algebraic Expressions and
Order of Operations

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. If $A \diamond B = \frac{A + B}{\sqrt{B}}$, evaluate $(6 \diamond 9) \diamond 25$.

2. Following order of operations, find the value of

$$24 \div 6 \times 2 - 7 \div \frac{1}{2} + 3 \times 5.$$

3. If $x \oslash y = 3y - 4x$, $x \bowtie y = xy \oslash y$, and $x \circ y = y \oslash (x - y)$, evaluate $(3 \circ 4) \bowtie 2$.

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____

(3 pts.) 3. _____



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Round 2: Solving Linear Equations

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Solve for x : $4 - (2 - 3x) = 6x - (4x + 3)$.

2. Solve for y :

$$\frac{\frac{y-3}{3} - 3}{\frac{3}{3}} = 11.$$

3. Solve for z :

$$\frac{3}{4}z + \frac{5}{6} = \frac{7}{8}z + \frac{9}{10}.$$

ANSWERS

(1 pt.) 1. $x =$ _____

(2 pts.) 2. $y =$ _____

(3 pts.) 3. $z =$ _____



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Round 3: Logic Problems

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. Alice won't take part in the play if Betty is in it. Charles will only play if Alice is in it. The producer insists that only one of the girls be in the play. Two people are needed. Who is in the play?

2. Before it is cooked, a meat roast may weigh from 3 to 5 pounds. When cooked for a certain time, the roast may lose from 30% to 50% of its weight. Find the maximum and minimum number of pounds in the total weight of 4 roasts after cooking.

3. Five students each have a favorite food. No two of them have the same favorite. Adrienne's favorite is not pizza. Bill hates pizza and hamburgers. Cindy's favorite is french fries. One of the students' favorites is chicken wings, but that student is not Darryl. Eddie's favorite is onion rings. What is Adrienne's favorite food?

ANSWERS

(1 pt.) 1. _____ and _____

(2 pts.) 2. max: _____ lb min: _____ lb

(3 pts.) 3. _____



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Round 4: Ratio, Proportion, and Variation

All answers must be in simplest exact form in the answer section

NO CALCULATOR ALLOWED

1. A recipe for a dozen cupcakes requires $2\frac{2}{3}$ cups of flour. How many cups of flour are needed to make 54 cupcakes?

2. The ratio of A to B is $4 : 5$. The ratio of B to C is $3 : 2$. The product of B and C is $k \cdot A^2$. Find k .

3. Suppose the variable E varies directly as the square of n and inversely as the product of m and L^2 . If $E = 96$ for $(n, m, L) = (4, 2, 1/4)$, find the positive value of n if $(E, m, L) = (81, 3, 1/6)$.

ANSWERS

(1 pt.) 1. _____ cups

(2 pts.) 2. _____

(3 pts.) 3. _____



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TEAM ROUND

All answers must either be in simplest exact form or rounded to EXACTLY three decimal places, unless stated otherwise. (3 POINTS EACH)

APPROVED CALCULATORS ALLOWED

1. The average of a set of nine numbers is 20. If the number 50 is added to the set, what is the average of the ten numbers?
2. In a subtraction problem, the 4-digit number $\underline{48xx}$ is subtracted from the four-digit number $\underline{197x}$, where the x's represent hidden digits, not necessarily identical. Find the greatest possible result obtainable.
3. Evaluate the expression
 $(1901 + 1902 + 1903 + \dots + 1999) - (1001 + 1002 + 1003 + \dots + 1099)$.
4. Given that the expression $\left(a(a(a)^2)^2\right)^2$ can be simplified to a^n , find n .
5. The ratio of boys to girls in the freshman class is 3 : 4. If $1/8$ of the girls are absent and $1/12$ of the boys are absent, then 125 freshmen will be in school. How many students are in the freshman class?
6. If it takes 5 sips to make 2 swallows, 7 swallows to make 3 swigs, and 4 swigs to make 3 gulps, how many sips are in 18 gulps?
7. Orville's and Wilbur's piggy banks each contain the same amount of money, all in coins. Orville has 3 times as many nickels as he has dimes and has 3 more quarters than dimes. Pennies make up half of his coins. Wilbur has 5 fewer dimes than he has nickels, 7 fewer pennies than dimes, and 4 more quarters than pennies. If Orville and Wilbur have the same number of nickels, how much money is in Orville's bank?
8. A number x is decreased by twenty percent. The result is increased by $1/3$ of itself. If that result is 4 more than x , what is the value of x ?



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TEAM ROUND ANSWER SHEET

1. _____

2. _____

3. _____

4. _____

5. _____ students

6. _____ sips

7. \$ _____

8. _____



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ANSWERS

ROUND 1

(Bancroft, Auburn, Hudson Catholic)

1. 6
2. 9
3. 158

ROUND 2

(Bromfield, Worc Acad, Doherty)

1. -5
2. 336
3. $-8/15 = -0.5\bar{3}$

ROUND 3

(Bromfield, Doherty, Shepherd Hill)

1. Alice and Charles (either order)
2. max: 14; min: 6
3. hamburgers

ROUND 4

(Auburn, Worc Acad, QSC)

1. 12
2. $25/24 = 1\frac{1}{24} = 1.041\bar{6}$
3. 3

TEAM ROUND

(Worc Acad, South, Algonquin, Hudson, Quaboag, Assabet Valley, Assabet Valley, Douglas)

1. 23
2. -2821
3. 89100
4. 14
5. 140
6. 140
7. \$3.53
8. 60



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FULL SOLUTIONS

ROUND 1

1. We have that $(6 \diamond 9) \diamond 25 = 5 \diamond 25 = \boxed{6}$.
2. Remember that multiplication and division are equally prioritized in the order of operations and they are to be performed left to right.

$$\begin{aligned} & 24 \div 6 \times 2 - 7 \div \frac{1}{2} + 3 \times 5 \\ &= 4 \times 2 - 14 + 15 \\ &= \boxed{9}. \end{aligned}$$

(Note that $24 \div 6 \times 2 = 8$ and not $24 \div 12 = 2$. Also, $7 \div \frac{1}{2} = 14$, not 3.5.)

3. First, compute $3 \circ 4 = 4 \oslash -1 = -3 - 16 = -19$. Then, $-19 \bowtie 2 = -38 \oslash 2 = 6 + 152 = \boxed{158}$.

ROUND 2

1. Remember to distribute the negative sign through the parentheses.

$$\begin{aligned} 4 - (2 - 3x) &= 6x - (4x + 3) \\ 2 + 3x &= 2x - 3 \\ x &= \boxed{-5}. \end{aligned}$$

2. Work the problem backwards. Multiply both sides by 3, add 3 to both sides, etc. Repeat the process three times to find that $y = \boxed{336}$.
3. The lcm of the denominators is 120, so multiply through to clear fractions.

$$\begin{aligned} 120 \left[\frac{3}{4}z + \frac{5}{6} \right] &= \frac{7}{8}z + \frac{9}{10} \\ 90z + 100 &= 105z + 108 \\ -8 &= 15z \\ \boxed{-\frac{8}{15}} &= z \end{aligned}$$



ROUND 3

- For two people, the possibilities are A&B, B&C, and A&C. Only A&C (Alice and Charles) is not ruled out by the problem's conditions.
- The maximum weight is $4 \cdot 5 \cdot 0.7 = \boxed{14}$ (largest roasts, smallest loss in weight). The minimum is $4 \cdot 3 \cdot 0.5 = \boxed{6}$ (smallest roasts, largest loss in weight).
- Organize the given information in a grid as follows:

	Piz	Ham	FF	Ch	OR
A	x		x		x
B	x	x	x		x
C	x	x	Y	x	x
D			x	x	x
E	x	x	x	x	Y

Darryl is the only student who can have pizza as his favorite. Therefore, his favorite is not hamburgers. This leaves Adrienne to like hamburgers and Bill to like chicken wings.

ROUND 4

- Our proportion gives $\frac{8}{3} \cdot \frac{54}{12} = \frac{8 \cdot 9}{3 \cdot 2} = \boxed{12}$.
- METHOD I:** We know that $B = \frac{5}{4}A$. Also, $C = \frac{2}{3}B = \frac{2}{3} \cdot \frac{5}{4}A = \frac{5}{6}A$. Therefore, $B \cdot C = \frac{5}{4} \cdot \frac{5}{6}A^2$, so $k = \frac{5}{4} \cdot \frac{5}{6} = \boxed{\frac{25}{24}}$.
METHOD II: Use the lcm to find the extended ratio $A : B : C = 12 : 15 : 10$. Therefore, $B \cdot C = 150$ and $A^2 = 144$. This gives $k = 150/144 = \boxed{25/24}$.
- We are given that $E \propto \frac{n^2}{mL^2}$, or that $\frac{EmL^2}{n^2} = k$ for some constant k . Plugging in the given values, we find that $k = \frac{3}{4}$. Now, plug in the new values and find that $n^2 = 9$. The problem asks for the positive value of n , so $n = \boxed{3}$.

TEAM ROUND

- The sum of the first nine numbers is $9 \cdot 20 = 180$. Adding 50, the sum of the ten numbers is $180 + 50 = 230$. The average is $230/10 = \boxed{23}$.



2. To maximize the result, we want to minimize $\underline{48xx}$ and maximize $\underline{197x}$. Therefore, choose $1979 - 4800 = \boxed{-2821}$.
3. Regroup the subtraction problem as $(1901 - 1001) + (1902 - 1002) + \dots + (1999 - 1099) = 900 \cdot 99 = 900 \cdot (100 - 1) = \boxed{89100}$.
4. Evaluate from the inside out: $\left(a(a(a^2)^2)\right)^2 = \left(a(a^3)^2\right)^2 = \left(a^7\right)^2 = a^{14}$. Therefore, $n = \boxed{14}$.
5. Let the freshman class consist of $3x$ boys and $4x$ girls, with the value of x to be determined. If $1/8$ of the girls and $1/12$ of the boys are absent, then the total number of students present is $(11/12)(3x) + (7/8)(4x) = 25x/4$. We are given that this is equal to 125 freshmen, so $x = 20$. Therefore, the size of the freshman class is $3x + 4x = 7x = \boxed{140}$.
6. Working backwards using ratios, 18 gulps equal 24 swigs. Since 3 swigs equal 7 swallows, 24 swigs equal 56 swallows. Finally, 2 swallows equal 5 sips, so 56 swallows equal $\boxed{140}$ sips.
7. Let P, N, D, Q be the number of pennies, nickels, dimes, and quarters, respectively, that Orville has. Similarly, let p, n, d, q be the number of pennies, nickels, dimes, and quarters, respectively, that Wilbur has.

From the given information, we know that

$$\begin{aligned} N &= 3D \\ Q &= D + 3 \\ P &= N + D + Q = 5D + 3. \end{aligned}$$

The total value, in cents, of these coins is $(5D + 3) + 15D + 10D + 25(D + 3) = 55D + 78$.

For Wilbur, we know that

$$\begin{aligned} n &= 3D \\ d &= n - 5 = 3D - 5 \\ p &= d - 7 = 3D - 12 \\ q &= p + 4 = 3D - 8. \end{aligned}$$

The total value of his coins is $(3D + 12) + 15D + 10(3D - 5) + 25(3D - 8) = 123D - 262$.

Since we are given that Orville and Wilbur have the same amount of money, $55D + 78 = 123D - 262$, so $D = 5$. Plugging back in, the total value of each person's coins is 353 cents, or $\boxed{\$3.53}$.



8. We have that

$$\frac{4}{3} \cdot \frac{4}{5}x = x + 4$$

$$\frac{16}{15}x = x + 4$$

$$\frac{1}{15}x = 4$$

$$x = \boxed{60}.$$