

WORCESTER COUNTY MATHEMATICS LEAGUE

Freshman Meet 2 – January 7, 2009

Round 1: Algebraic Word Problems

1

All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1. Mary's mom is four times older than Mary. In four years she will be three times older than Mary. What is Mary's age now?

2. Find the largest of four consecutive integers such that the sum of the two largest subtracted from three times the sum of the two smallest is 70.

3. A large tank is filled to the top with a 16% solution of alcohol in water. Exactly 5 gallons of the solution are removed from the tank and replaced with pure water. This has the effect of reducing the alcohol percentage to 10%. Find the exact capacity of the tank, in gallons.

ANSWERS

(1 pt.) 1. _____ years old

(2 pts.) 2. _____

(3 pts.) 3. _____ gallons

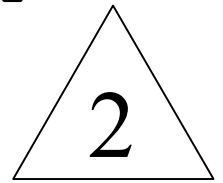
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Round 2: Number Theory

All answers must be in simplest exact form

NO CALCULATOR ALLOWED



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1. Find the greatest common factor of 189 and 294.

 2. Convert the sum $11_2 + 111_3 + 1111_4$ to a single base-5 number. (Note: the subscript indicates the number's base.)

 3. How many positive integer factors does the base ten number 3,600 have?

ANSWERS

(1 pt.) 1. _____

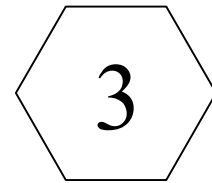
(2 pts.) 2. _____

(3 pts.) 3. _____

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Round 3: Operations on Fractions, Decimals, Percents
and Percentage Word Problems



All answers must be in simplest exact form

NO CALCULATOR ALLOWED

1. If 1 is increased by 10% and that result is then decreased by 10%, what value is produced?

2. At the auditions for the school musical, 30% of the students were eliminated after the first week. Of those remaining, 20% were chosen. If seven students were chosen, how many students auditioned for the musical?

3. Evaluate the following expression in simplest form:

$$\left(\frac{0.3 + \frac{1}{6}}{\frac{2}{5} - 0.6} \div \frac{1}{\frac{2}{5}} \right)^2$$

ANSWERS

(1 pt.) 1. _____

(2 pts.) 2. _____ students

(3 pts.) 3. _____

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Round 4: Set Theory

4

All answers must be in simplest exact form

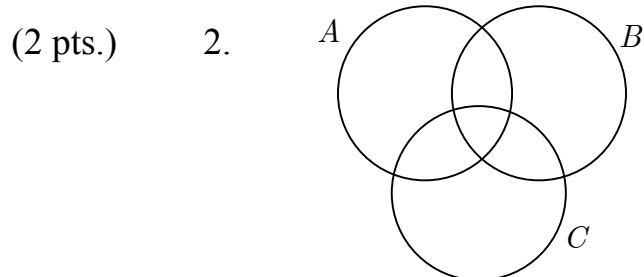
NOTE: S' indicates the complement of the set S

NO CALCULATOR ALLOWED

1. In my pre-calculus class 12 students are taking chemistry, 10 are taking physics, and 3 are taking both chemistry and physics. If 5 students are not taking either chemistry or physics, how many students are in my pre-calculus class?
2. Using the Venn diagram in the answer section below, clearly shade the region represented by the set $((A \cup B) \cap C) \cap A'$.
3. If $A = \{0, 1, 3, 5, 6, 7, 8\}$, $B = \{\text{all positive even integers}\}$ and $C = \{5, 6, 7, 8, 9, 10, 11, 12\}$, how many subsets containing exactly two elements are there of the set that results from $(A \cup B) \cap C$?

ANSWERS

(1 pt.) 1. _____ students



(3 pts.) 3. _____

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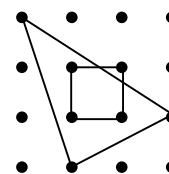
TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

APPROVED CALCULATORS ALLOWED

1. Find all possible values of x that satisfy the equation $4 \cdot |2x| + 5 = 7 \cdot |2x| + 3$.
2. The elements of set B are all of the possible subsets of set A. If set B has 16 subsets, find the number of elements in set A.
3. If the nine-digit number $8K2,54K,K32$ is divisible by 72, find the value of the digit K .
4. Find the smallest counting number X such that $\overline{0.42} + \overline{0.221403} = \overline{0.X}$.
5. The difference between the squares of two consecutive integers is 199. Find the sum of the squares of these two consecutive integers.
6. Find the ordered pair (x, y) that satisfies the following pair of equations:
$$123x + 321y = 345$$
$$321x + 123y = 543$$
7. I always walk at a constant rate of 4 miles per hour and jog at a constant rate of 6 miles per hour. I find that I can save 3 minutes and 45 seconds by jogging, instead of walking, to school in the mornings. How far do I live from my school (in miles)?

8. In the grid to the right the dots are evenly spaced, 1 unit apart. Find the area of the region common to the triangle and the square.



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All answers must be in simplest exact form!

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ANSWER SHEET - TEAM ROUND

All answers must *either* be in simplest exact form *or* as decimals rounded correctly to at least three decimal places! (3 pts. each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. (____,____)

7. _____ miles

8. _____

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ANSWERS

Round 1

- 8
- 21
- $13\frac{1}{3} = \frac{40}{3} = 13.\overline{3}$

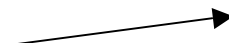
Round 2

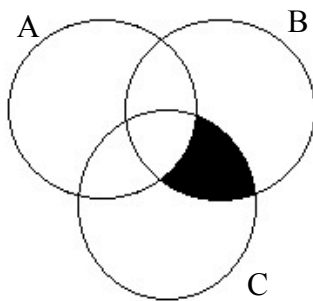
- 21
- 401 (or 401_5)
- 45

Round 3

- $0.99 = \frac{99}{100}$
- 50
- $\frac{9}{4} = 2\frac{1}{4} = 2.25$

Round 4

- 24
- 
- 15



Team Round

- $\frac{1}{3} = 0.\overline{3}$, $-\frac{1}{3} = -0.\overline{3}$
(need both & OK to accept answers rounded to at least 3 decimal places)

2. 2

3. 4

4. 645

5. 19,801

6. $\left(\frac{3}{2}, \frac{1}{2}\right) = \left(1\frac{1}{2}, \frac{1}{2}\right) = (1.5, 0.5)$

7. $\frac{3}{4} = 0.75$

8. $\frac{11}{12} = 0.9\overline{16}$

(or rounded correctly to at least 3 decimal places)